RBE 2004

ระบบอัตโนมัติ (Automatic System)

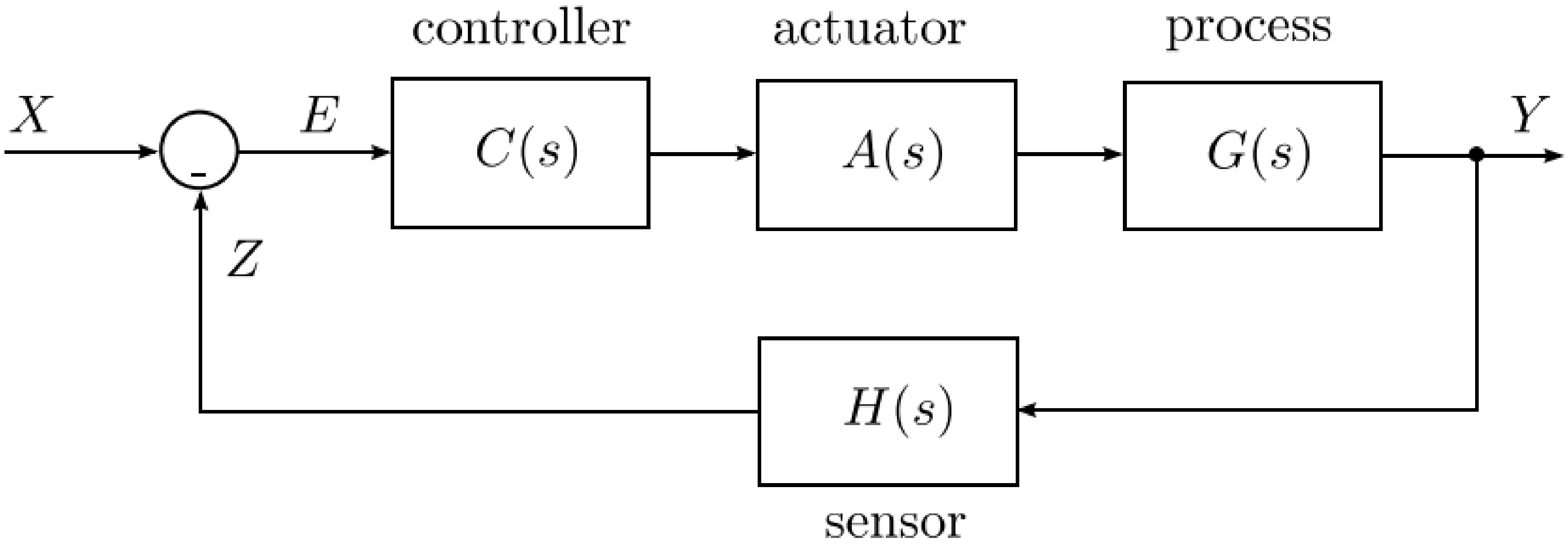
สาขาวิศวกรรมหุ่นยนต์
คณะวิศวกรรมศาสตร์และเทคโนโลยีอุตสาหกรรม
มหาวิทยาลัยราชภัฏสวนสุนันทา

Chapter 3 Characteristic of System
Lecture 5 Block Diagram and Steady State Error

- การเขียนระบบในรูป Block Diagram
  - การแสดงระบบควบคุมด้วย block diagram
  - ตัวอย่าง block diagram
  - การเขียน transfer function ของ closed – loop system
- การหาค่า Steady state error และการวิเคราะห์ผลตอบสนองของระบบภายใต้อินพุทแบบต่างๆ
- Computer Simulation (Matlab/Simulink)

Applications

What the transfer function of the closed-loop system shown ?



Applications

The position control system for a spacecraft platform is governed by the following equations:

$$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$

$$v_1(t) = r(t) - p(t)$$

$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$

$$v_2(t) = 8v_1(t)$$

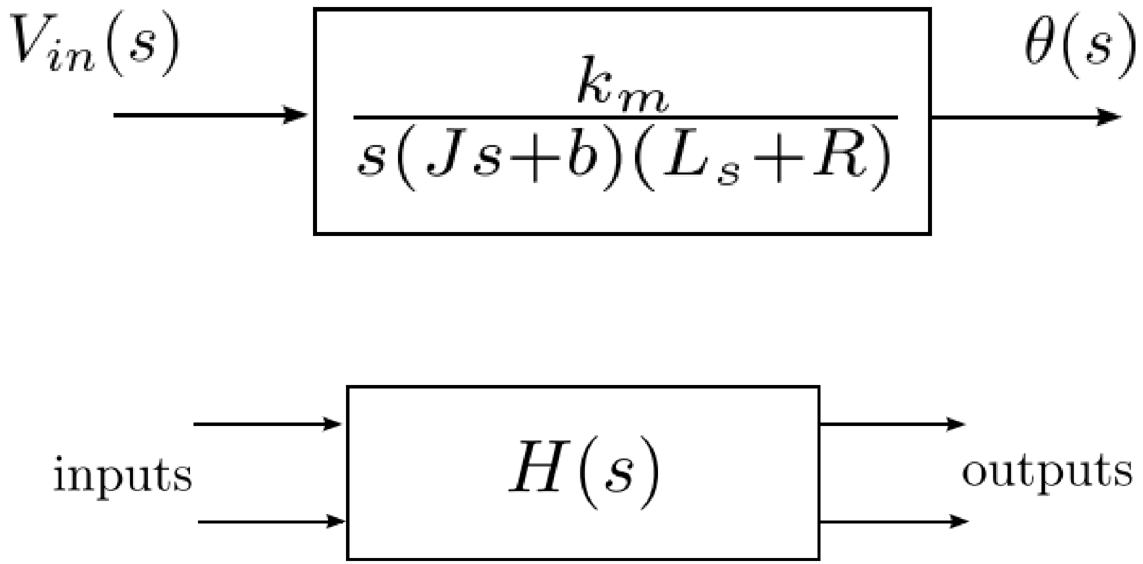
$r(t)$: desired position
$p(t)$: current position
$v_1(t)$: amplifier input voltage
$v_2(t)$: amplifier output voltage
$\theta(t)$: motor shaft position

How can we represent the system using a block diagram ?

Block diagrams

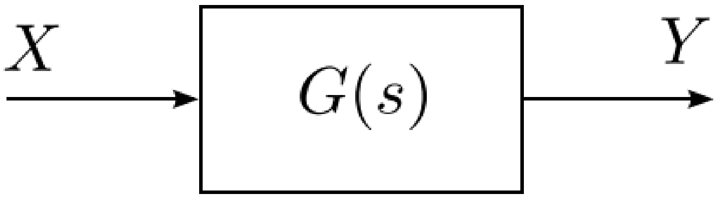
→ Represent the relationship of a system variables graphically.

→ Example: The relation between the input voltage and and the position of a DC motor

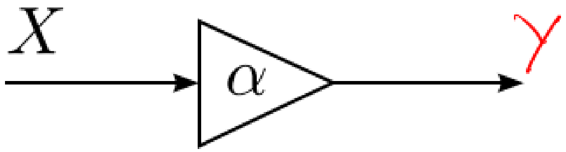


Basic building elements

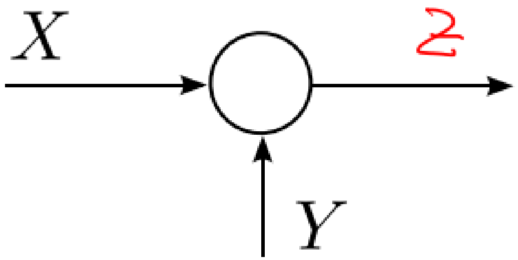
Transfer function



Gain

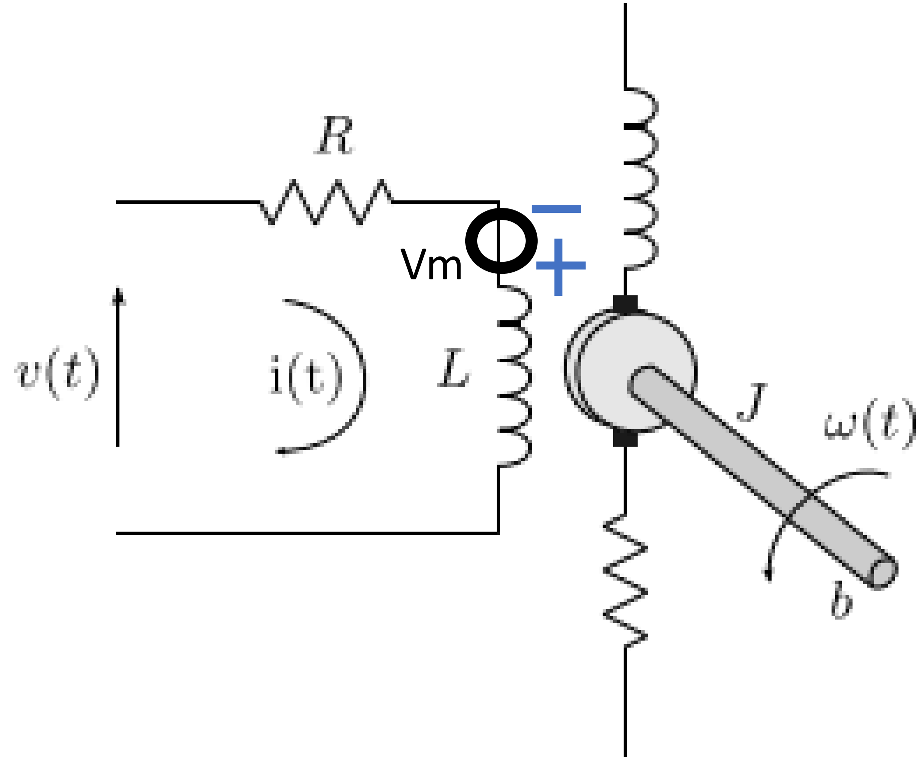


Sum



Block diagram of a DC motor

→ Electric circuit characteristics

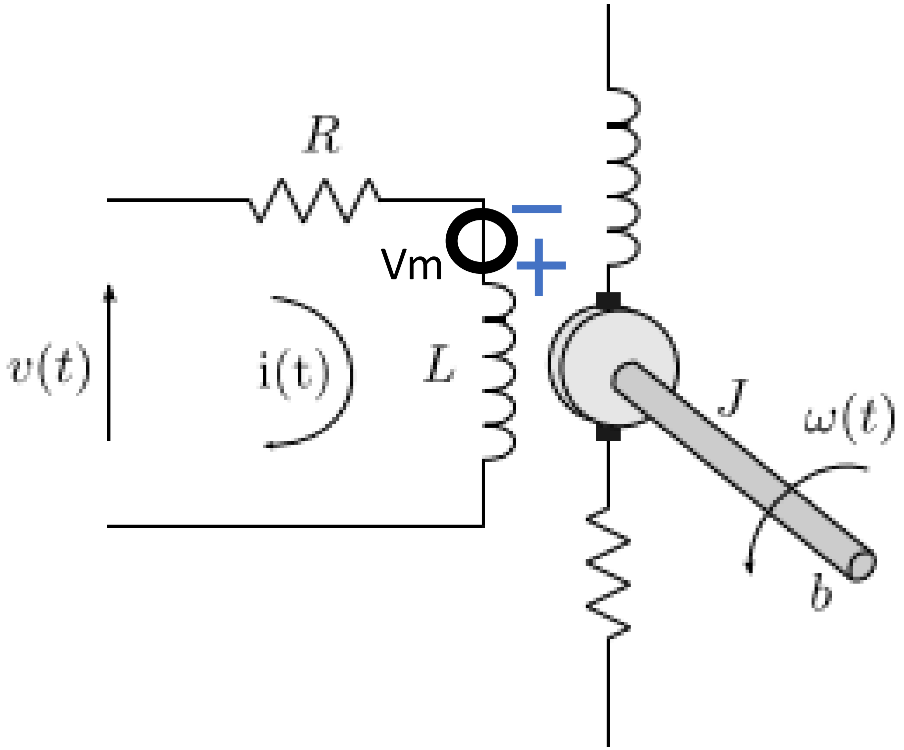


→ Back electromagnetic force voltage

$$V(s) = (R + Ls)I(s) + \omega(s)k_m \rightarrow I(s) = \frac{V(s) - V_m(s)}{R + Ls}$$

Block diagram of a DC motor

→ Mechanical characteristics



→ Torque constant

$$T(s) = (Js^2 + bs)\theta(s) + T_d \rightarrow \theta(s) = \frac{I(s)k_i - T_d}{Js^2 + bs} \rightarrow \omega(s) = \frac{I(s)k_i - T_d}{Js + b}$$

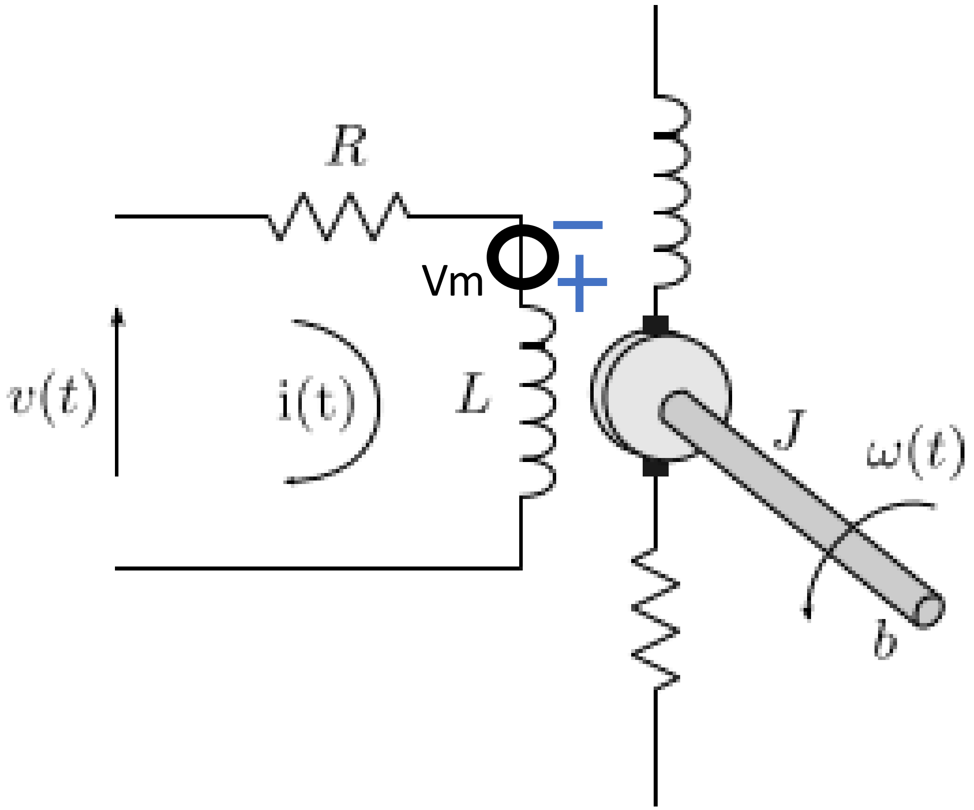
Block diagram of a DC motor

$$I(s) = \frac{V(s) - V_b(s)}{Ls + R}$$

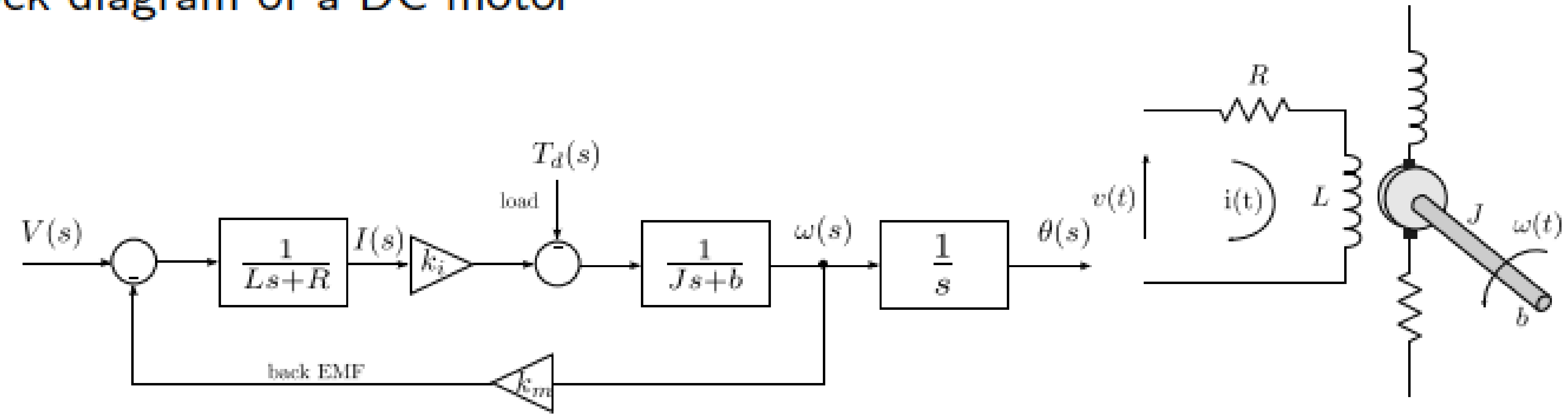
$$\omega(s) = \frac{T(s) - T_d(s)}{Js + b}$$

$$T(s) = k_i I(s)$$

$$V_m(s) = k_m \omega(s)$$



Block diagram of a DC motor

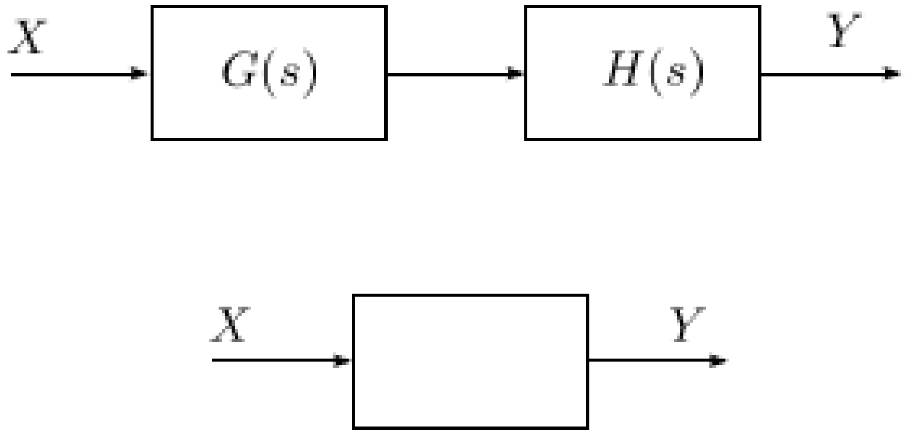


Simulation with Matlab - Simulink

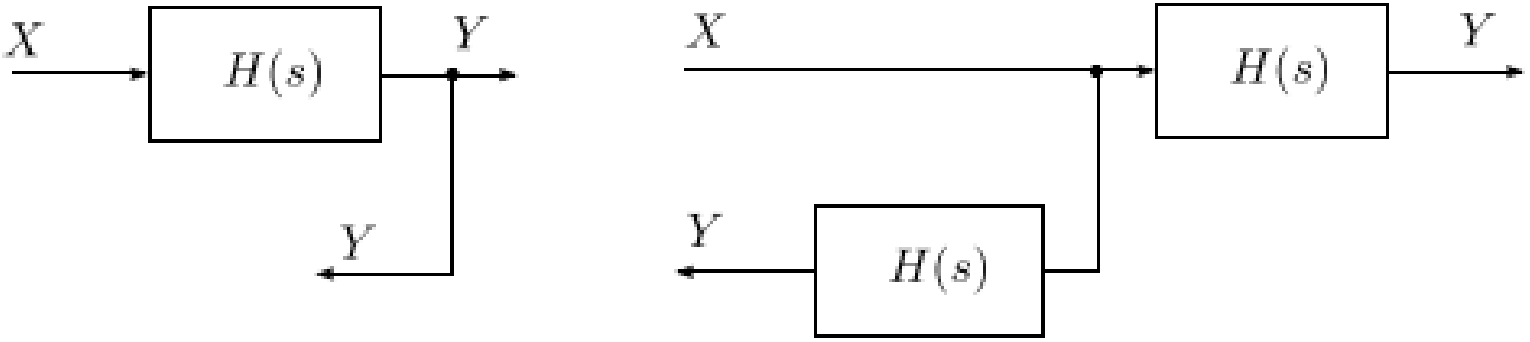
Evaluate the step response of the motor

Basic operations

Combining blocks in cascade

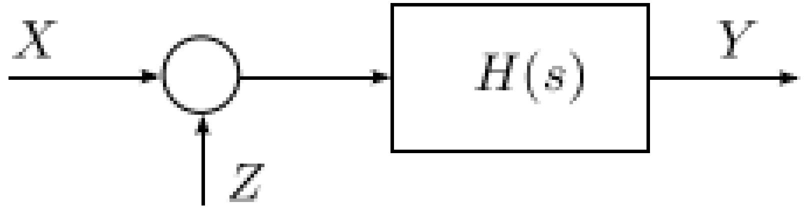


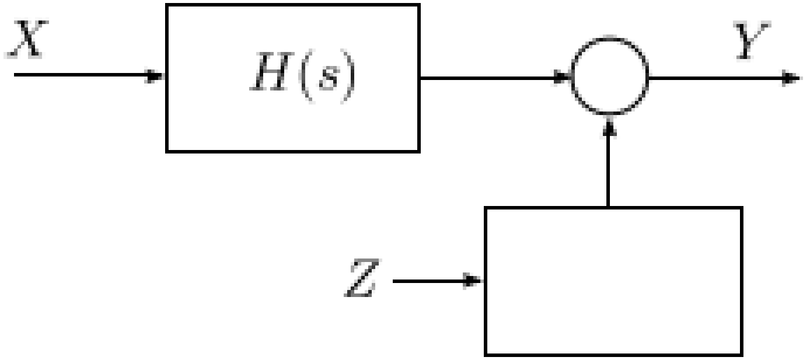
Moving a pickoff point ahead of a block



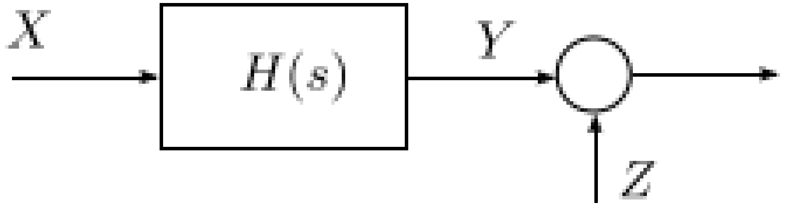
Basic operations

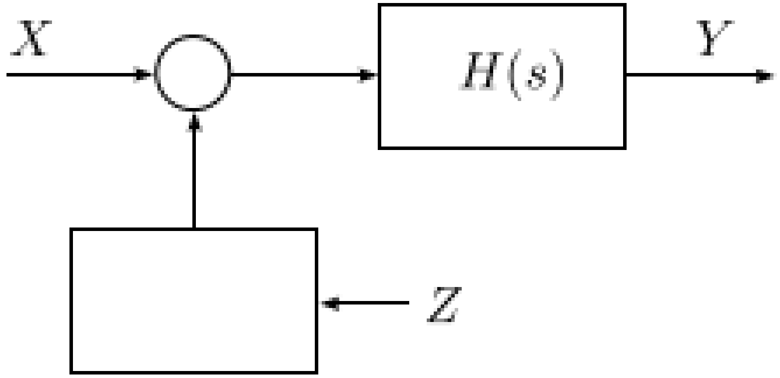
Moving a summing point ahead a block



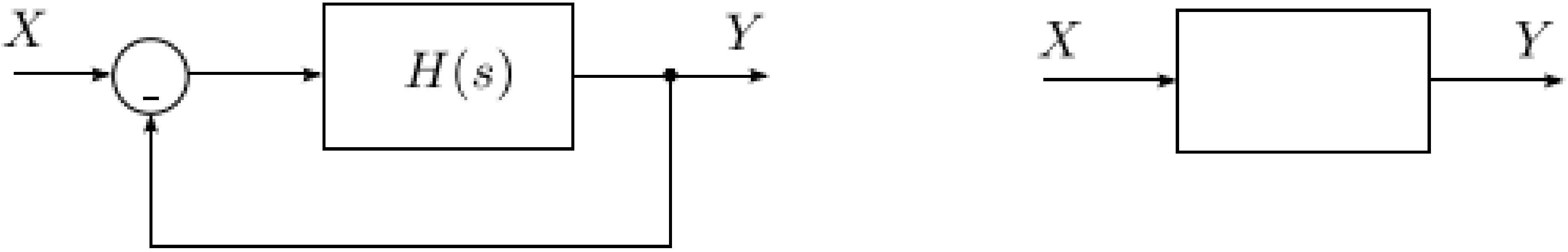


Moving a summing point behind of a block

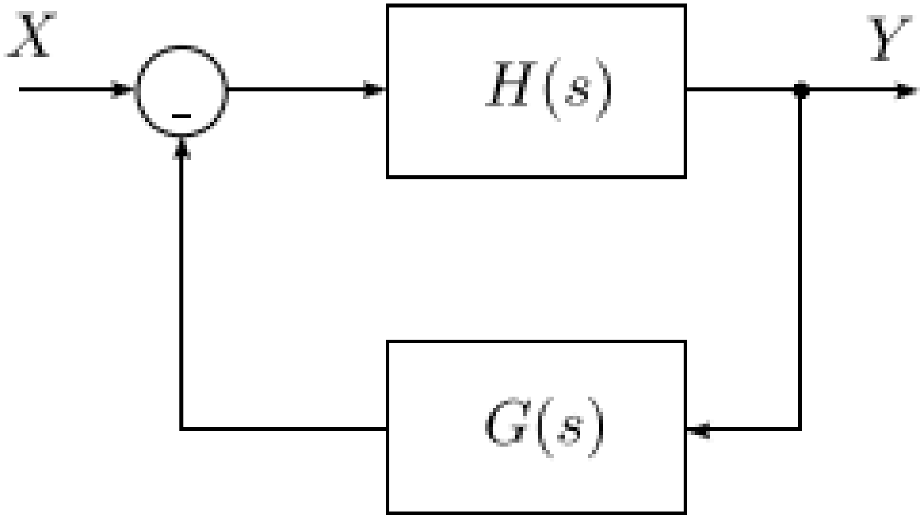




Eliminating a feedback loop

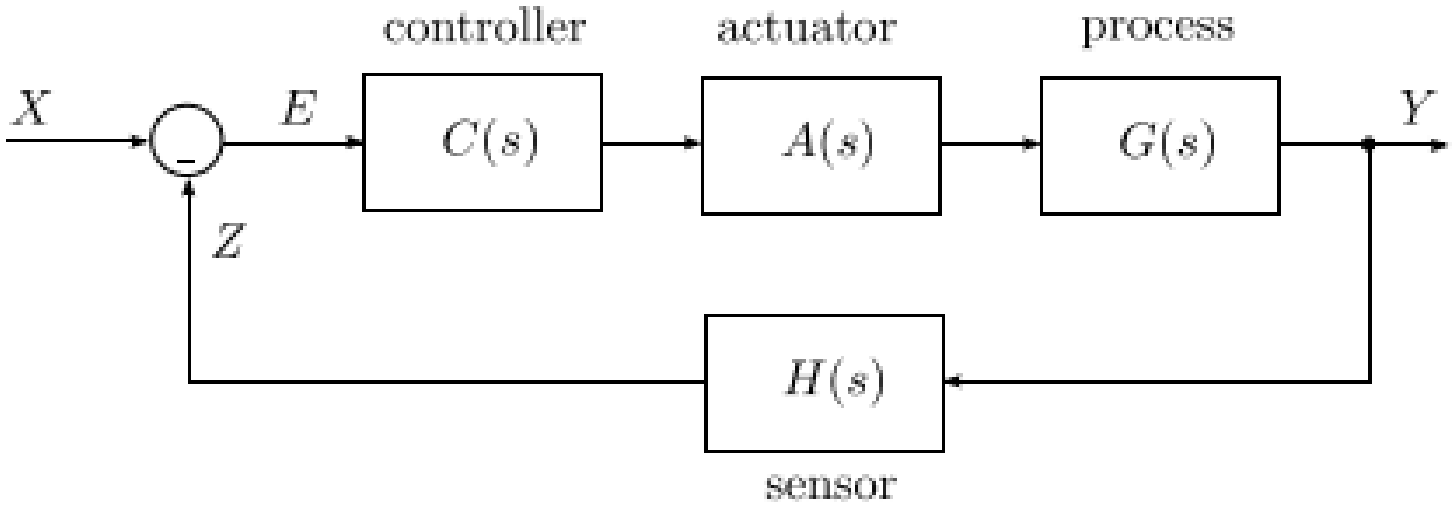


Eliminating a feedback loop

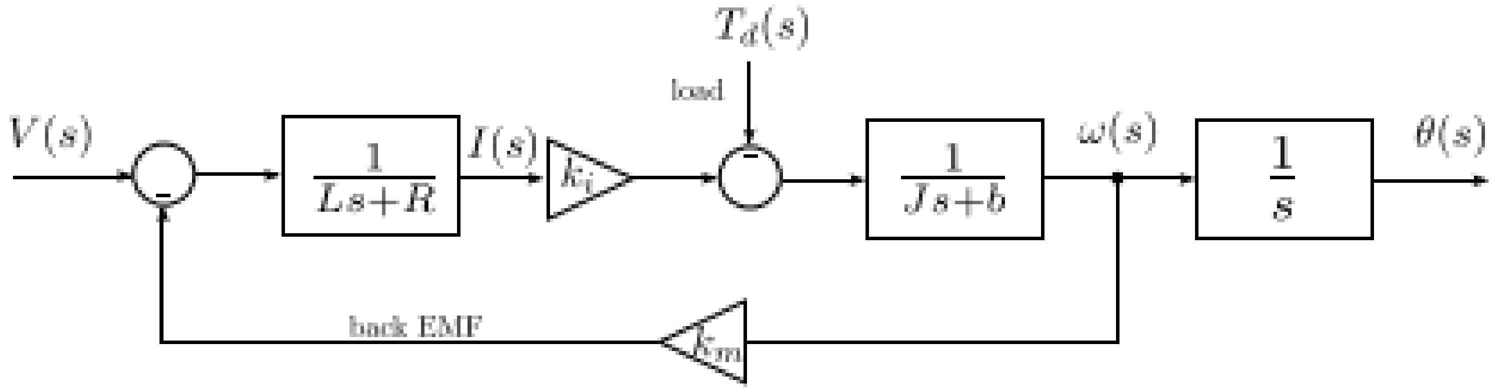


Example 1

Find the open-loop transfer function of the closed-loop system shown.

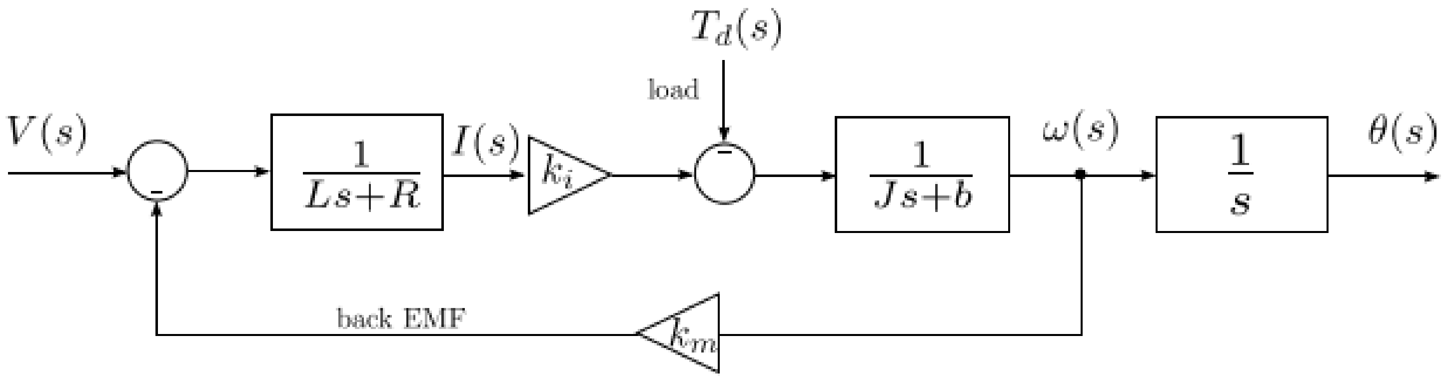


Example 2 - DC motor



If $T = 0$, what is the transfer function $\theta(s)/V(s)$?

Example 2 - DC motor



$$G(s) = \frac{\theta(s)}{V(s)} = \frac{k_i}{s[(Ls+R)(Js+b)+k_i k_m]} \tag{1}$$

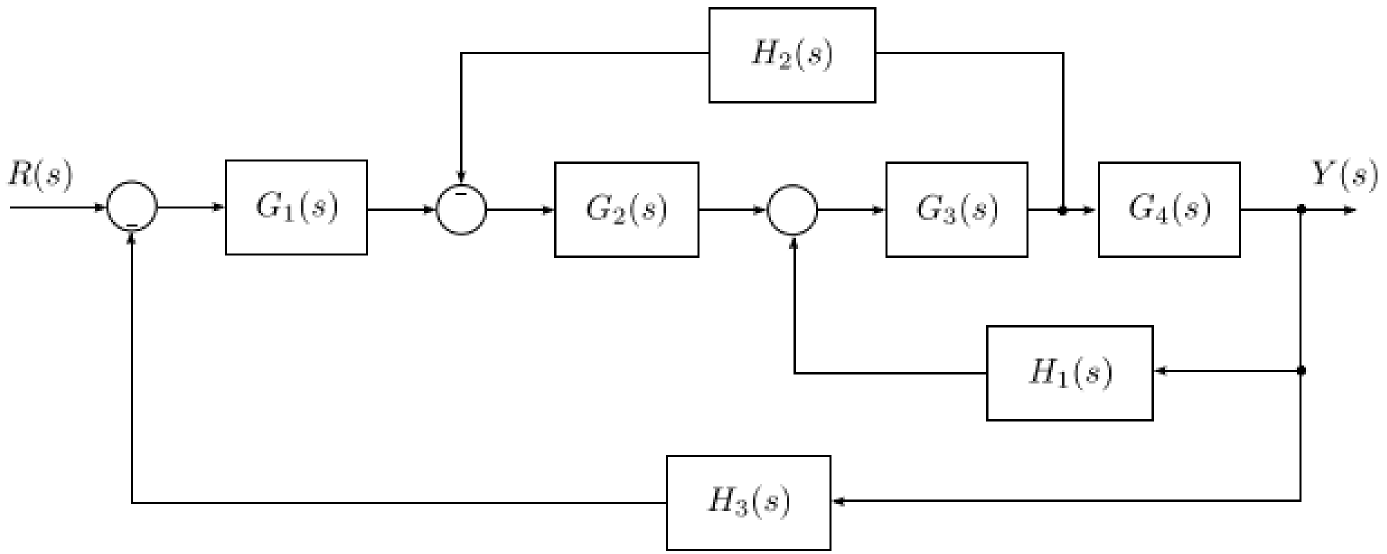
Sometimes the armature time constant $\tau_a = L/R$ is negligible, thus

$$G(s) \approx \frac{\theta(s)}{V(s)} = \frac{k_i}{s[R(Js+b)+k_i k_m]} = \frac{k_i/(Rb+K_i K_m)}{s(\tau s+1)} \tag{2}$$

where $\tau = \frac{RJ}{Rb+K_i K_m}$

Exercise 23

Find the transfer function $Y(s)/R(s)$ of the system shown.

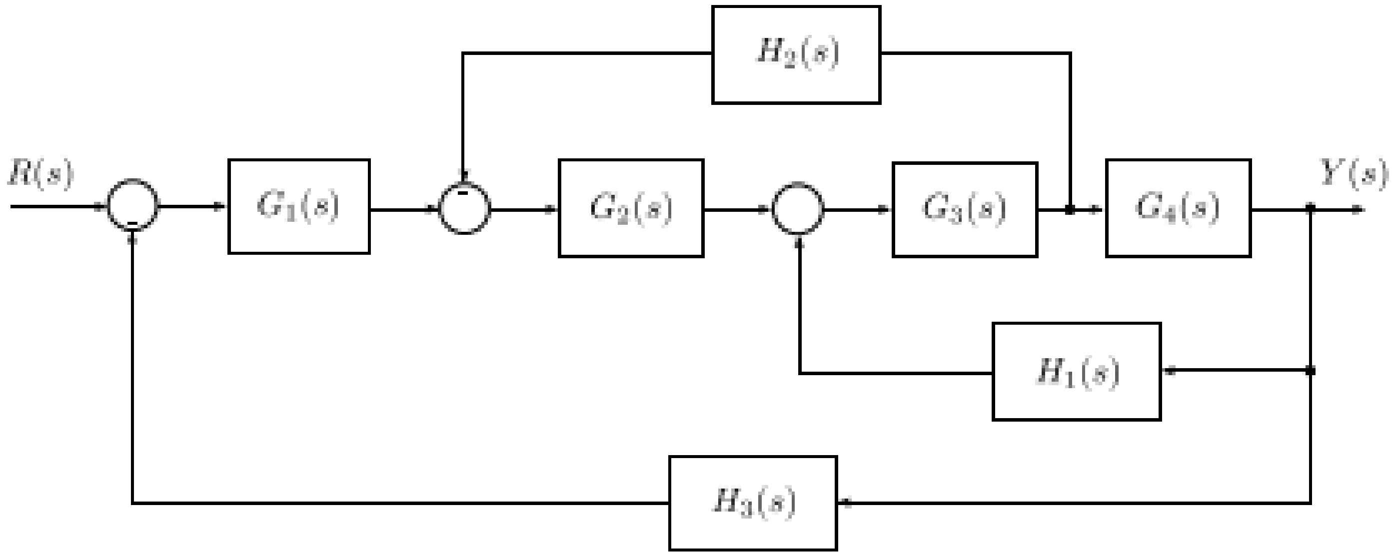


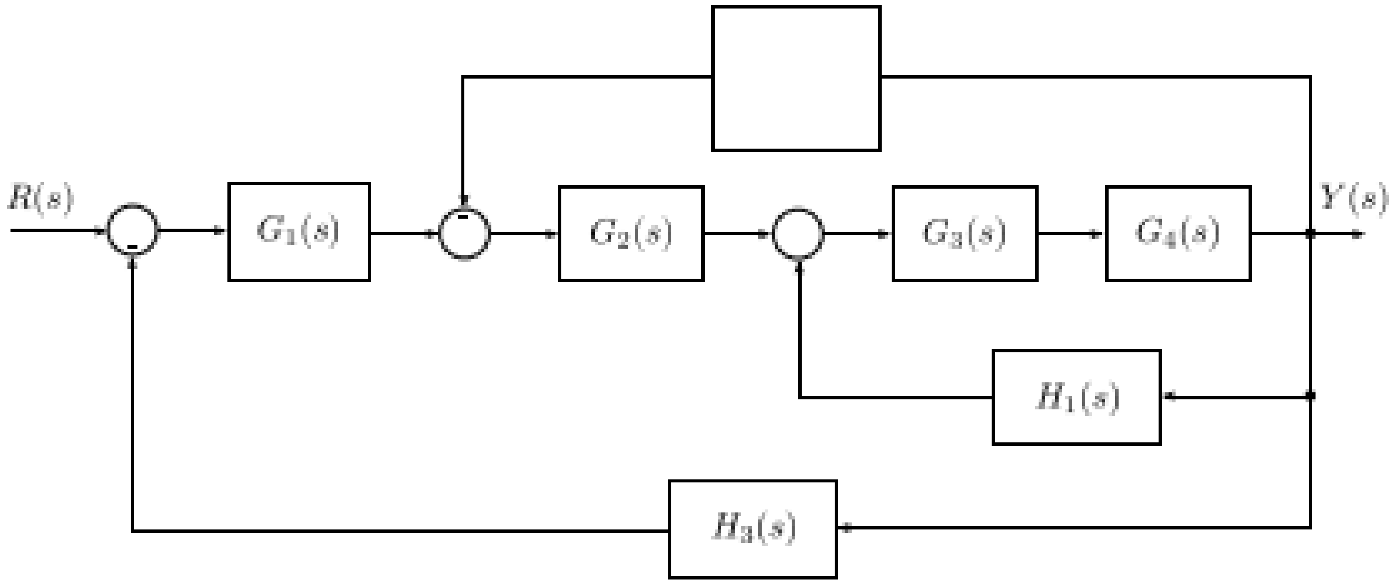
Procedure:

→ Simply the block diagram

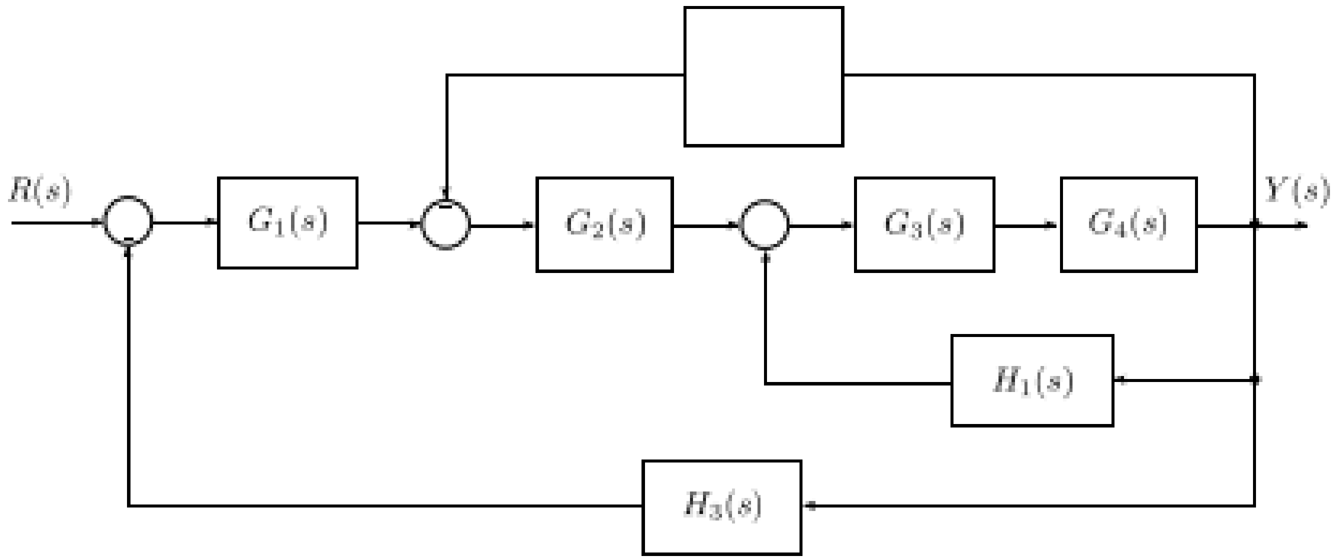
→ Calculate the closed-loop transfer function

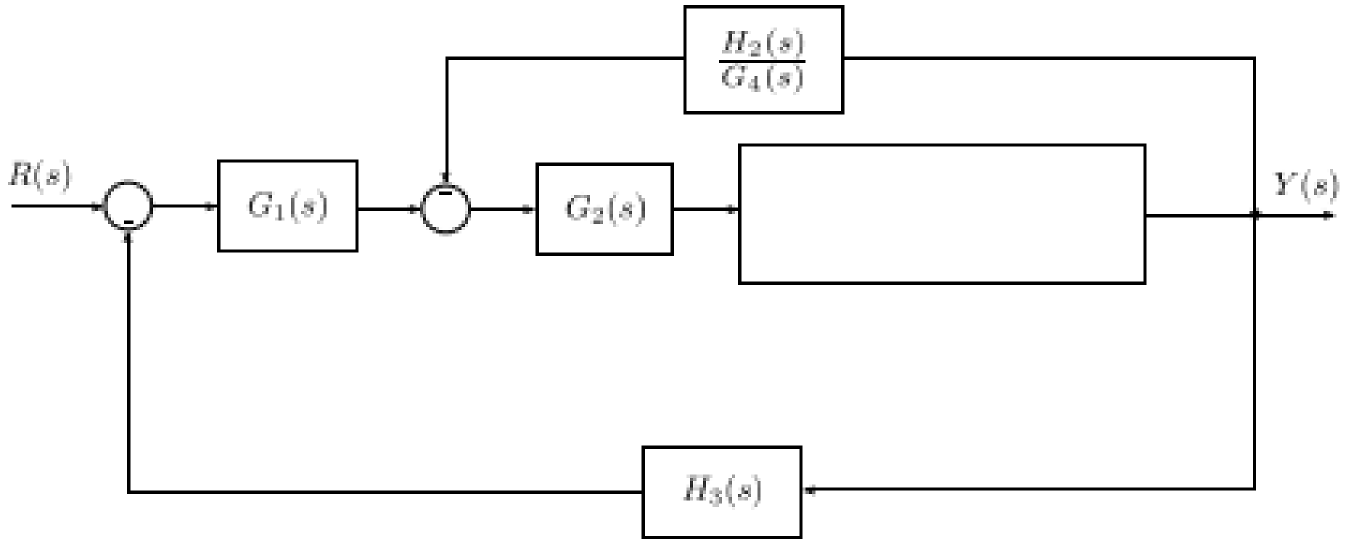
Exercise 23 - continued



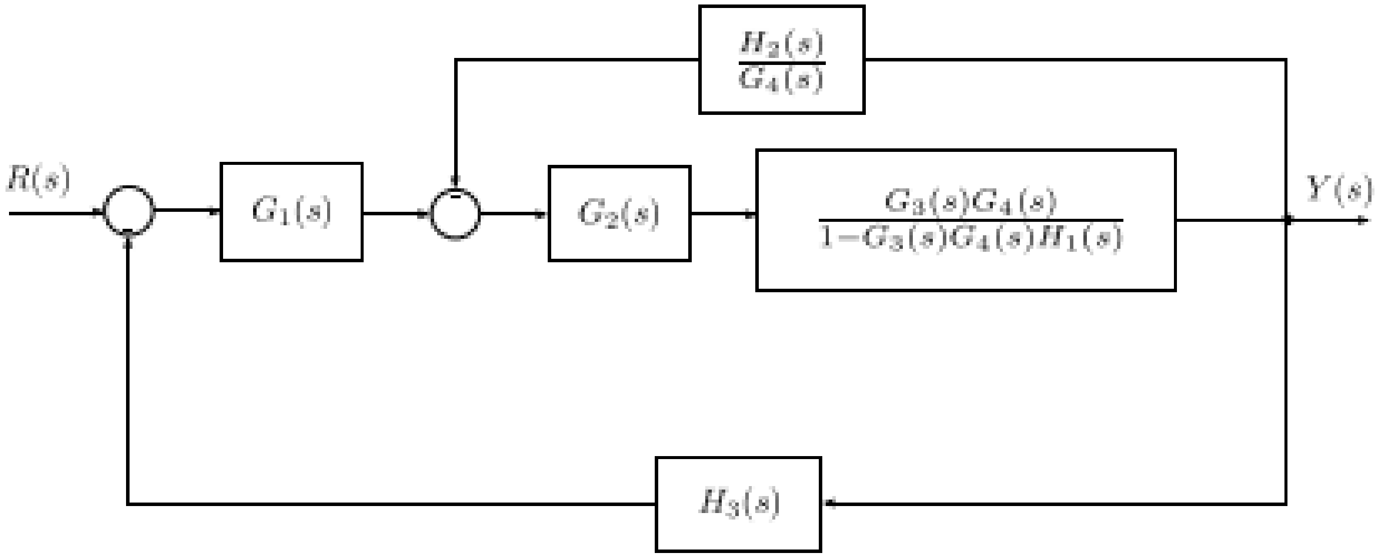


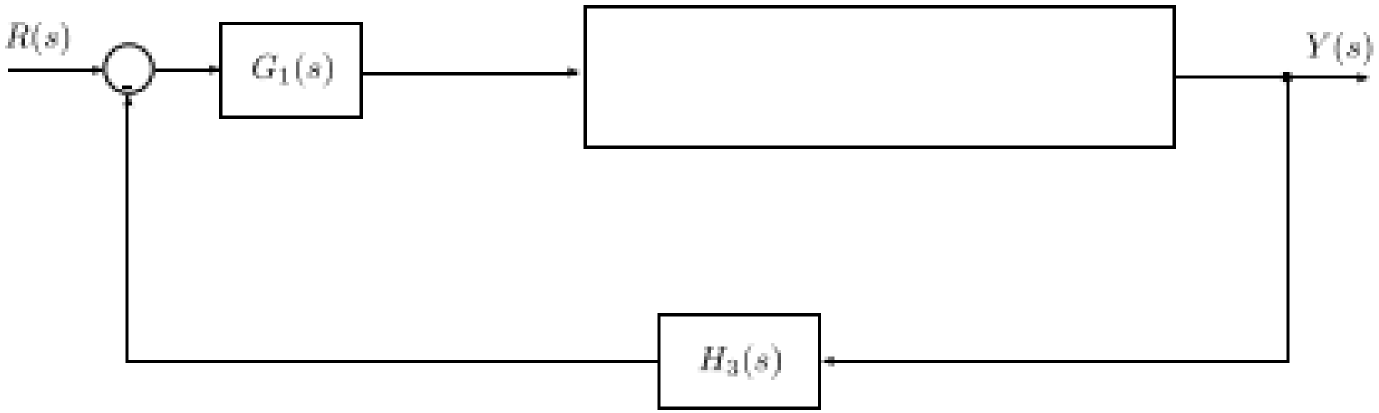
Exercise 23 - continued



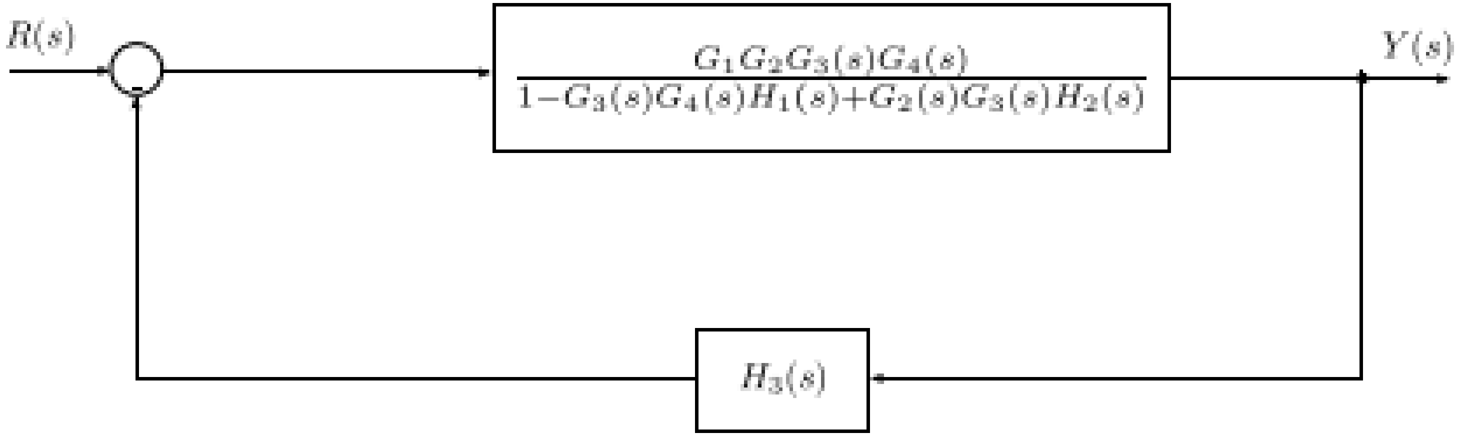


Exercise 23 - continued



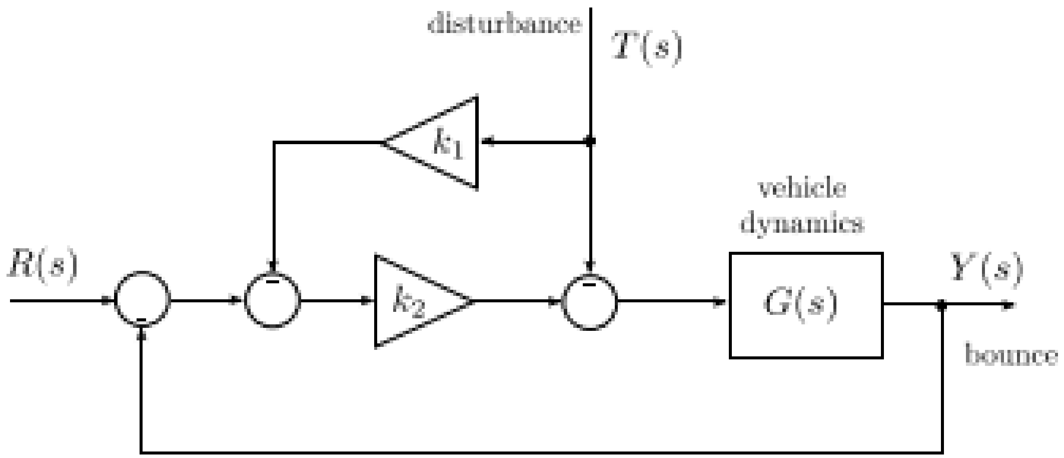


Exercise 23 - continued



Exercise 24

An active suspension system can be controlled by a sensor that looks ahead at the road conditions. An example that can accommodate road bumps is shown in the figure. Find the gain k_1 so that the vehicle does not bounce when the desired deflection is $R(s) = 0$ and the disturbance is $T(s)$.



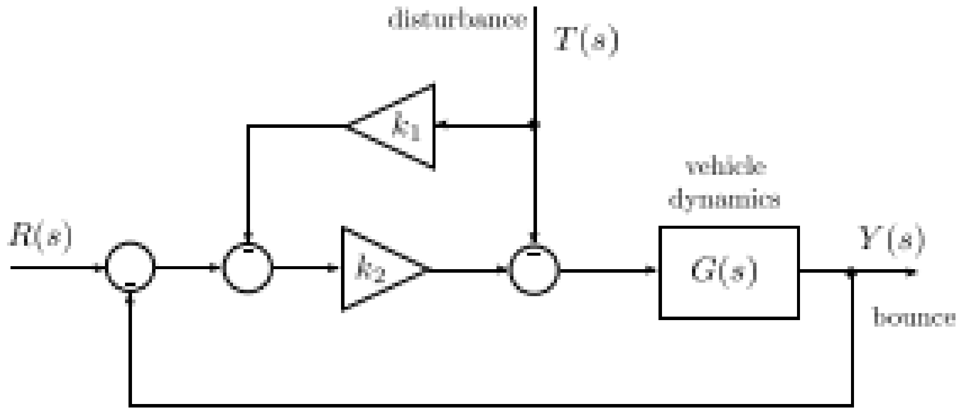
Procedure:

→ Find the transfer function from $T(s)$ to $R(s)$

→ Set the bounce to zero ($Y(s) = 0$)

→ Calculate k_1

Exercise 24 - continued



Exercise 25

The position control system for a spacecraft platform is governed by the following equations:

$$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$

$$v_1(t) = r(t) - p(t)$$

$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$

$$v_2(t) = 8v_1(t)$$

$r(t)$: desired position
$p(t)$: current position
$v_1(t)$: amplifier input voltage
$v_2(t)$: amplifier output voltage
$\theta(t)$: motor shaft position

To do:

→ Sketch a block diagram of the system

→ Find the transfer function $P(s)/R(s)$

Exercise 25 - continued

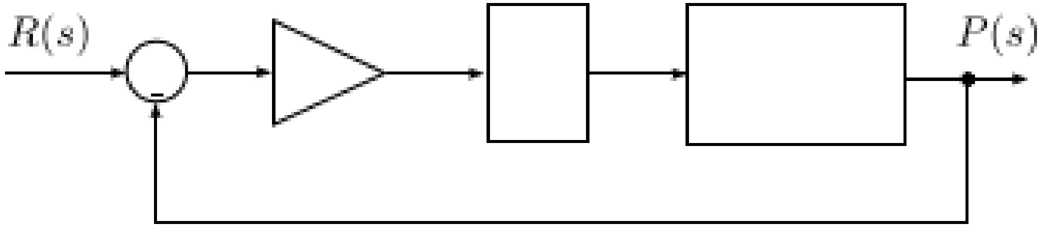
$$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$

$$v_1(t) = r(t) - p(t)$$

$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$

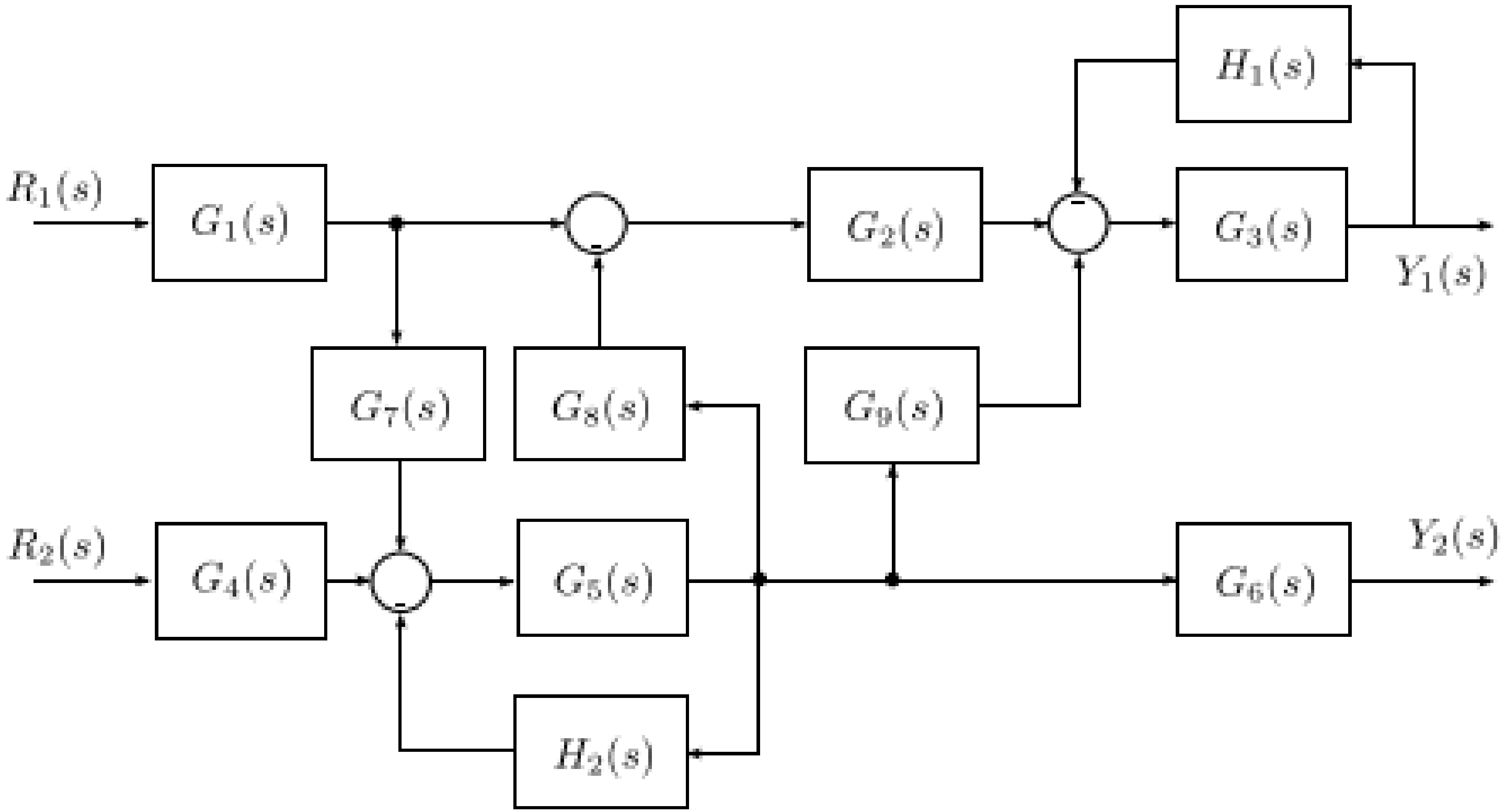
$$v_2(t) = 8v_1(t)$$

Exercise 25 - continued



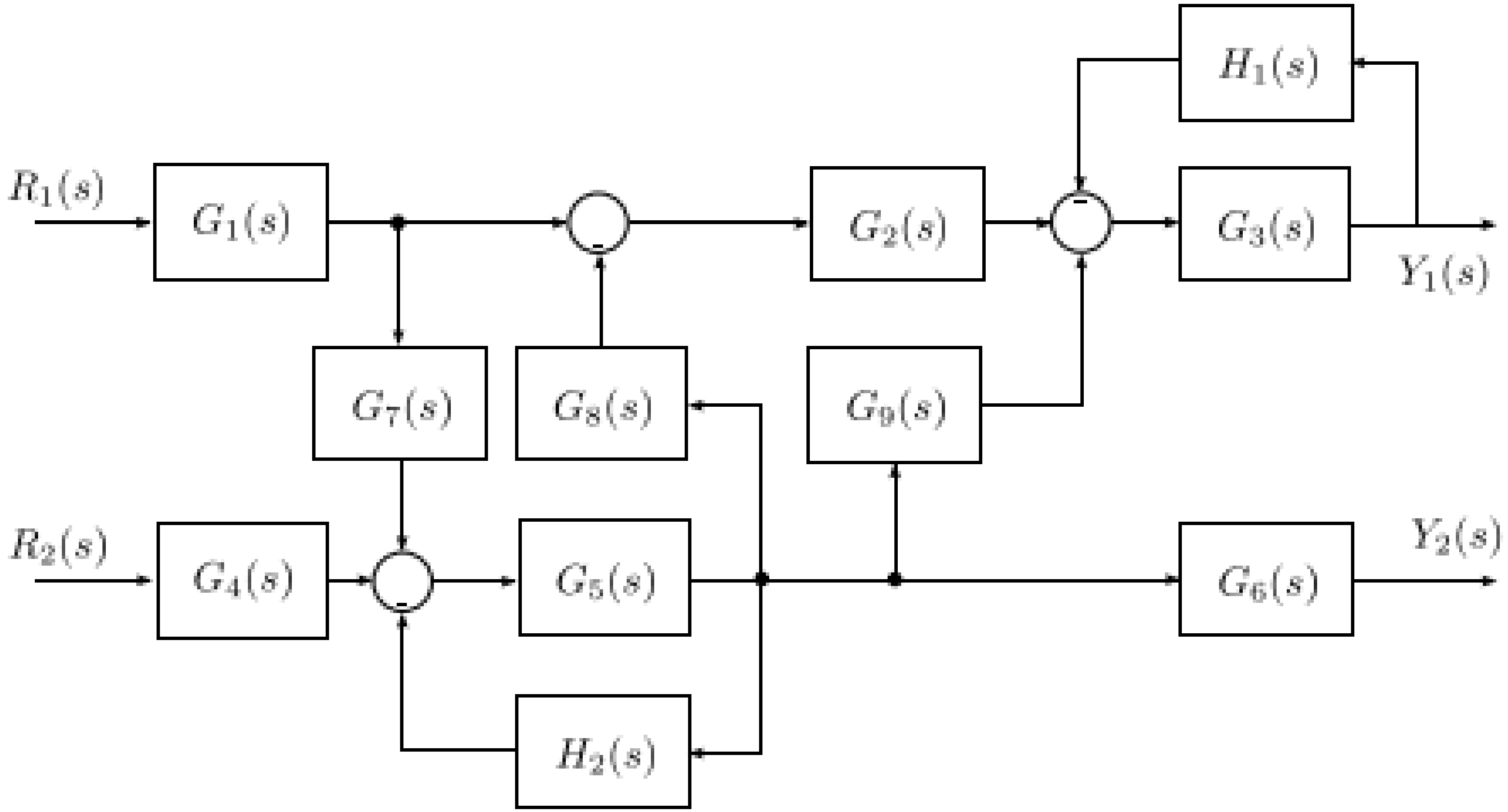
Exercise 26

Compute the transfer function $Y_1(s)/R_2(s)$. Hint: Using the principle of superposition, set $R_1(s) = 0$.



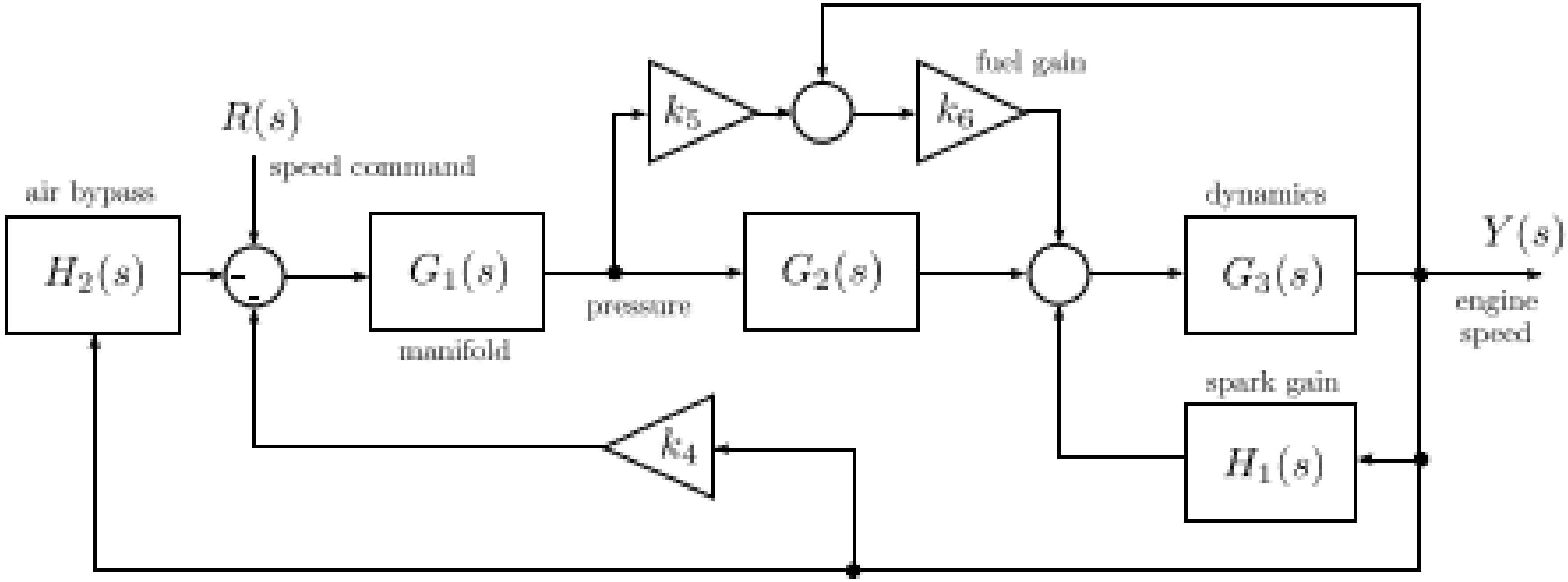
Exercise 27

Compute the transfer function $Y_2(s)/R_1(s)$. Hint: Using the principle of superposition, set $R_2(s) = 0$.



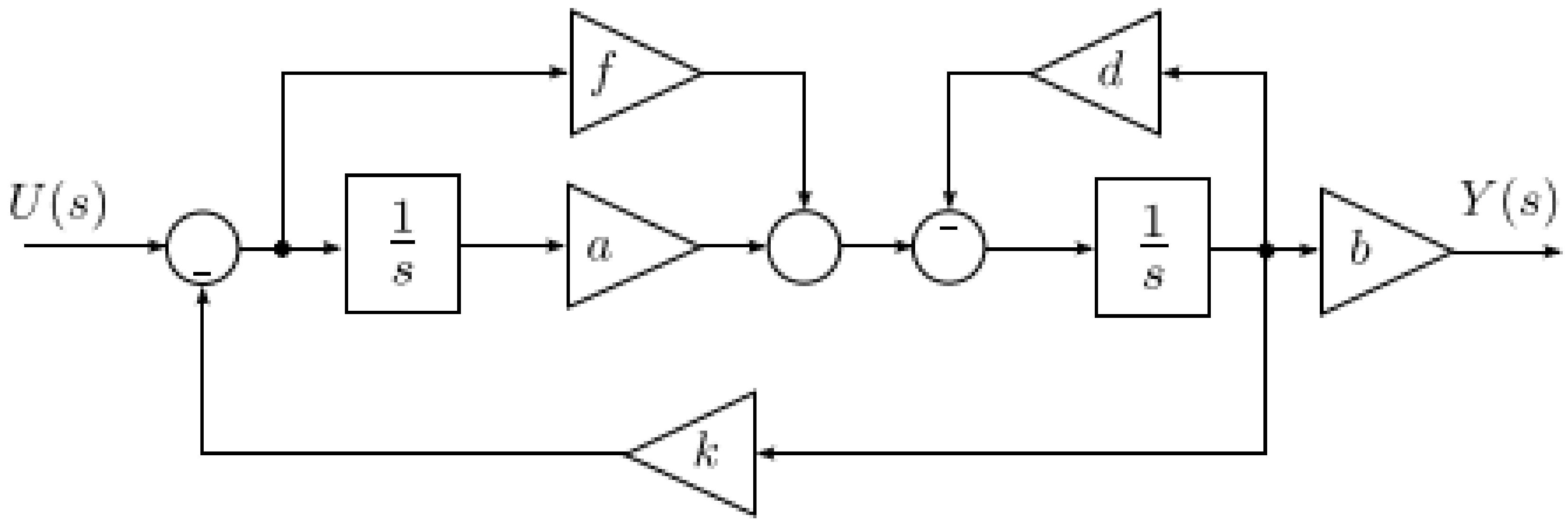
Exercise 28

Compute the transfer function $Y(s)/R(s)$ for the idle-speed control system for a fuel-injected engine as shown in the block diagram.



Exercise 29

Compute the transfer function $Y(s)/U(s)$ for the block diagram shown.

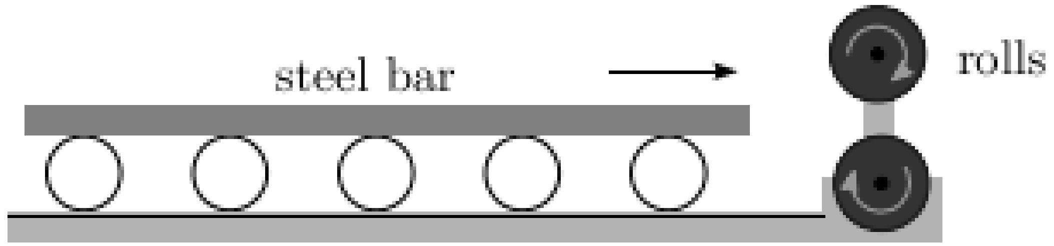


การหาค่า Steady state error และการวิเคราะห์ผลตอบสนองของระบบภายใต้อินพุทแบบต่างๆ

- Understand concept of error and disturbance signals
- Calculate the steady state error of a system
- Analyze the influence of control loop gain

Applications

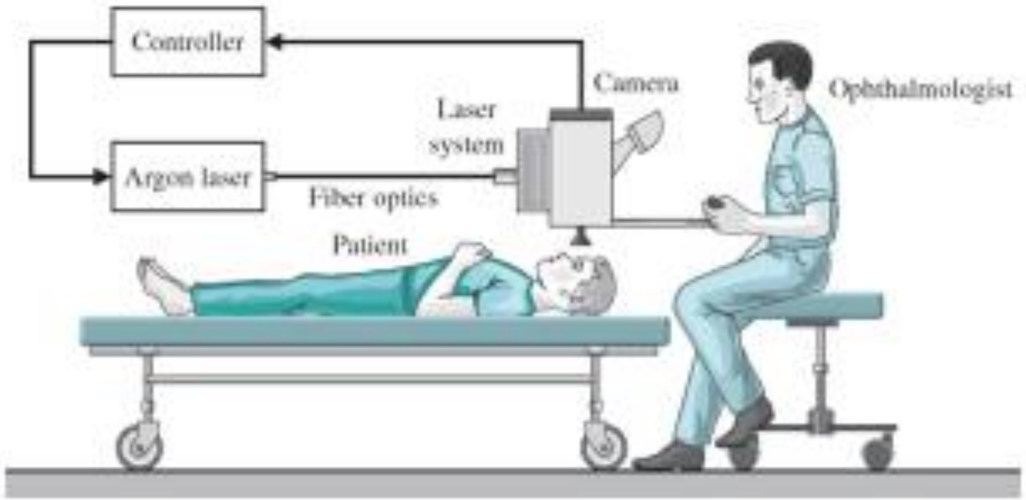
When the steel bars engage in the rolls of the rolling mill, the load on the rolls increases immediately. How can the speed of the rollers be controlled to minimize this disturbance?



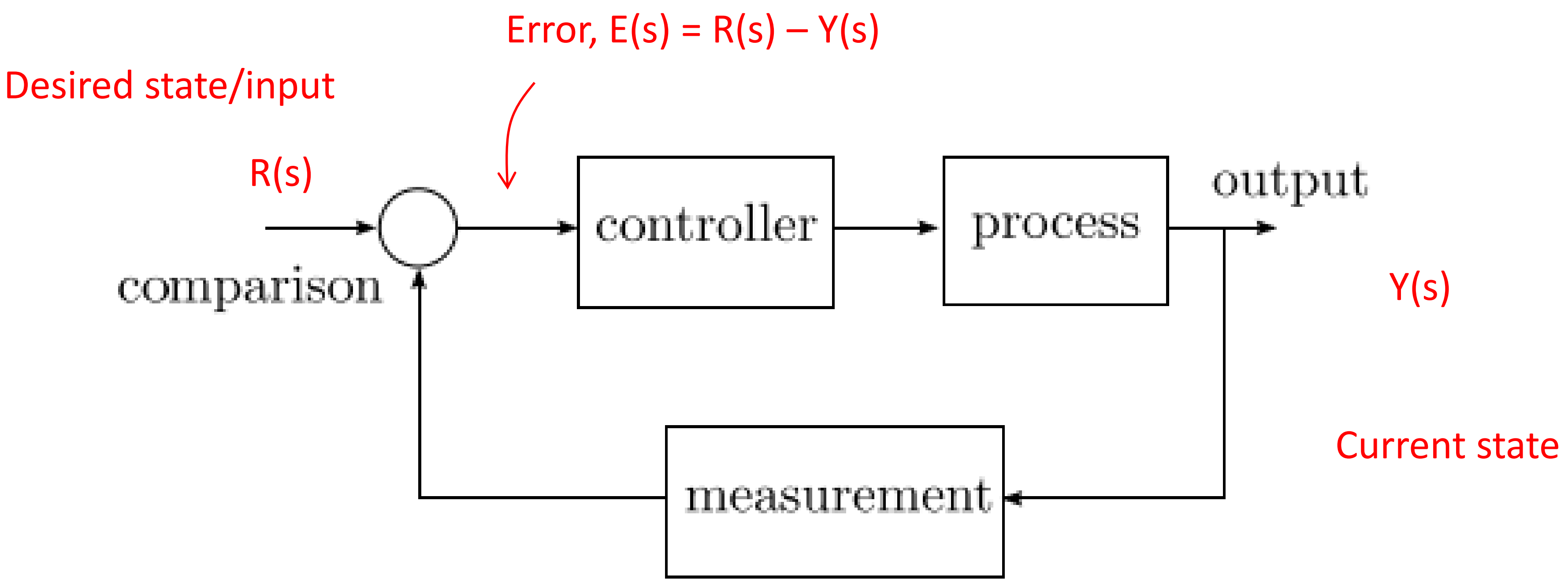
Applications

Automated control of the laser position during eye surgery enables the ophthalmologist to indicate to the controller where lesions should be inserted.

How can we design a controller that minimizes the transient response of the positioning system if the retina moves during the surgery?

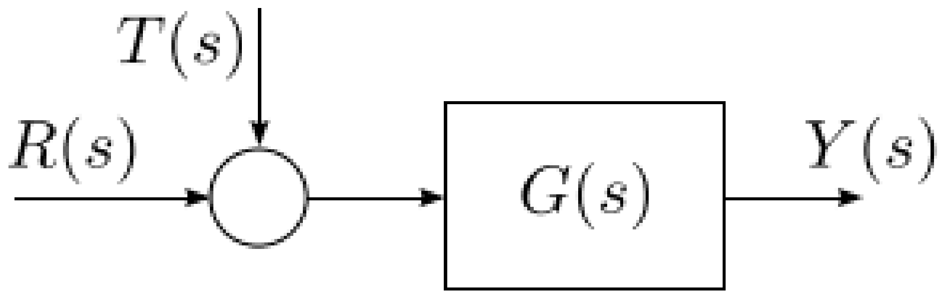


Closed-loop control

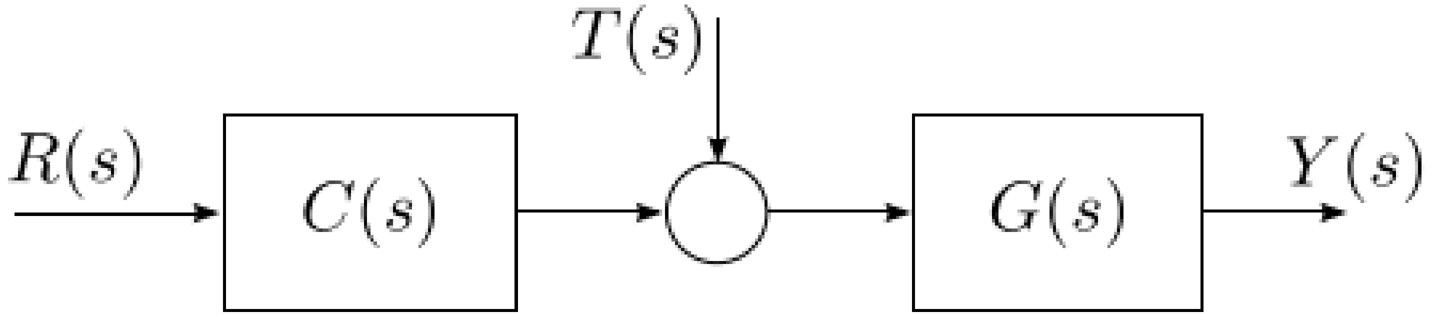


Open-loop control

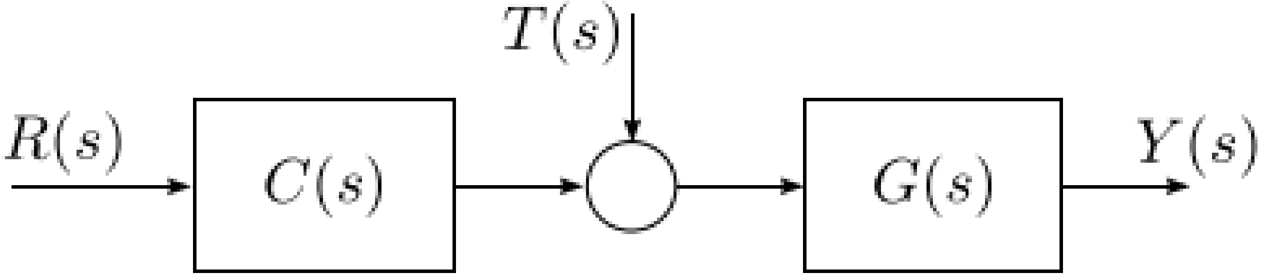
In the absence of a controller and without feedback, the disturbance $T(s)$ directly influences the output $Y(s)$.



An **open-loop** system operates without feedback and directly generates the output in response to an input signal.

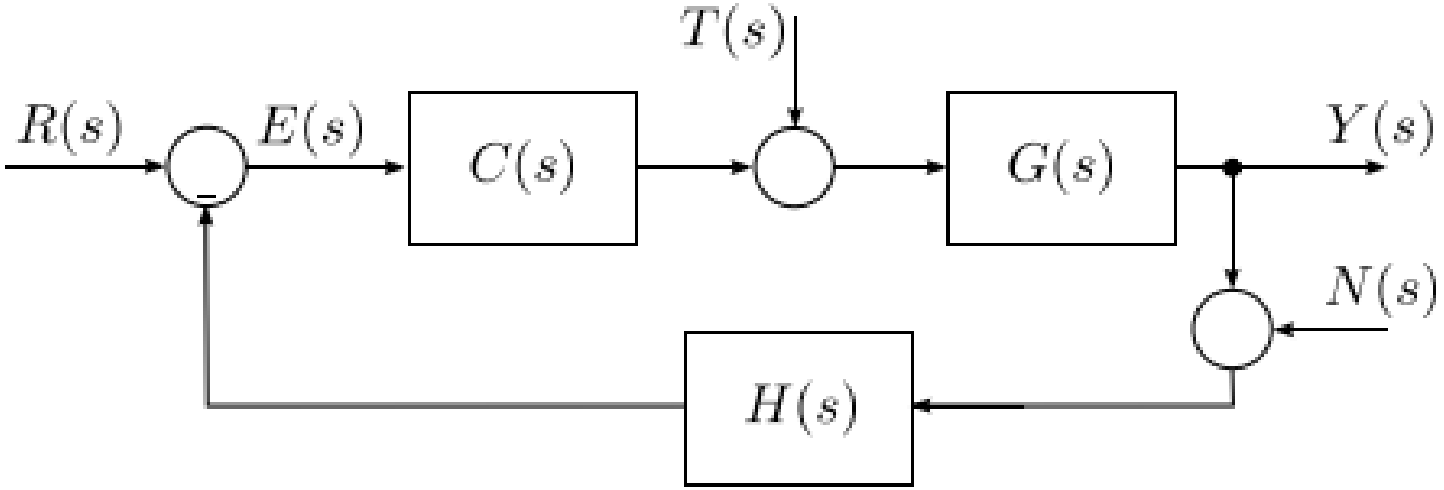


Closed-loop control

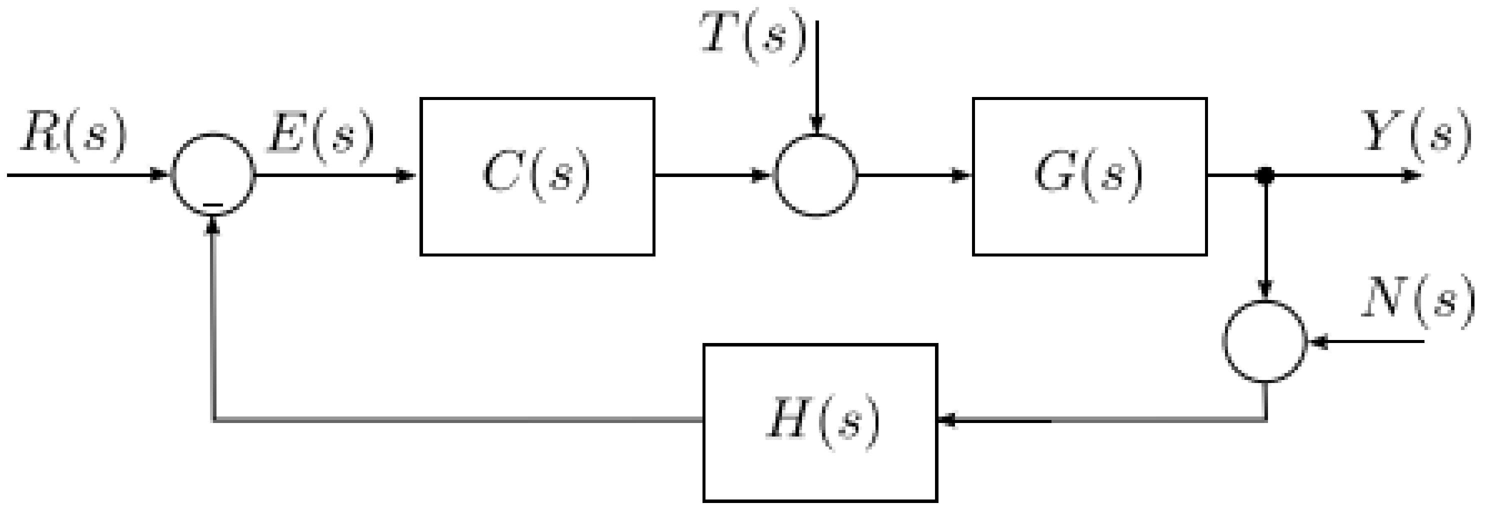


A **closed-loop** system compares the output $Y(s)$ with a desired value $R(s)$.

The error signal $E(s)$ is used by the controller to adjust the actuator.



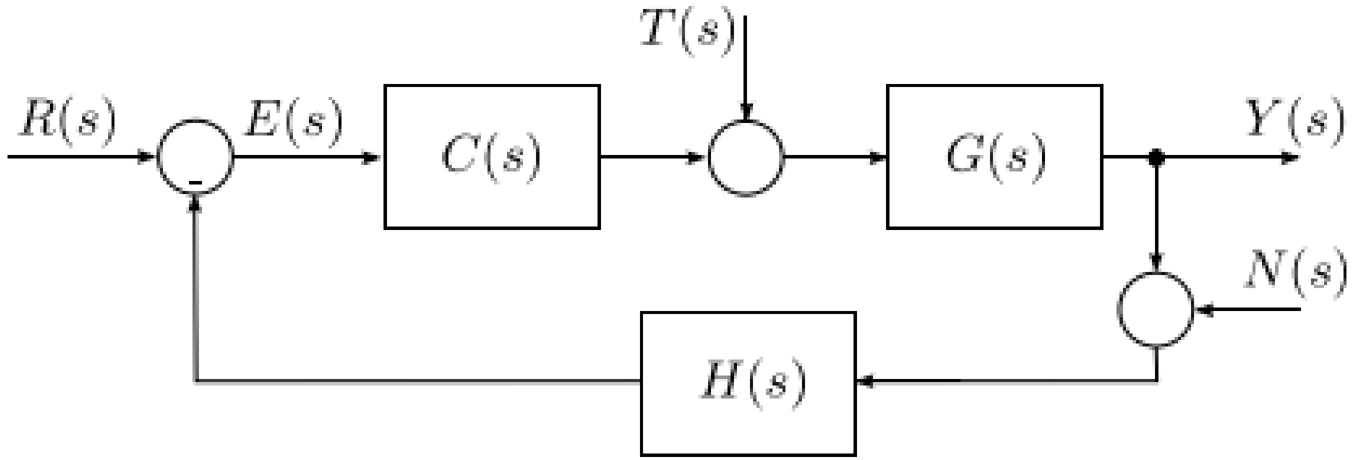
Advantages of closed-loop control



- Decrease sensitivity to variations in the parameters of the process ($G(s)$)
- Improve rejection of the disturbances ($T(s)$)
- Improve noise attenuation ($N(s)$)
- Reduce the steady-state error ($E(s)$)
- Allows for control of the transient response

Disturbance

Disturbance is a change in the values of the nominal parameters of a control system due to external sources.

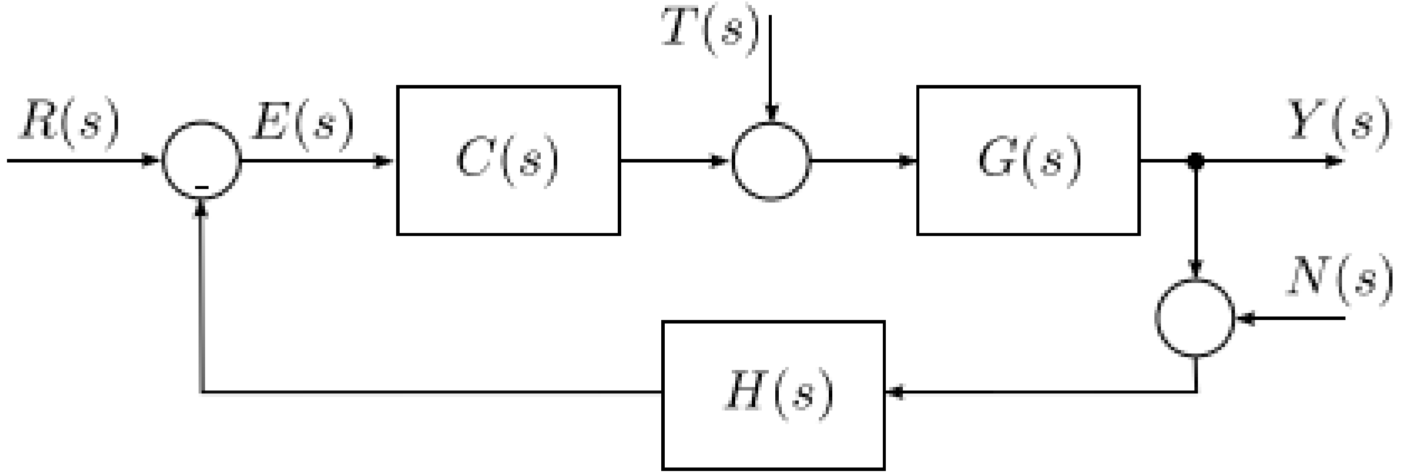


For $H(s) = 1$, the response due to disturbance is

$$Y_D(s) = \frac{G(s)}{1 + C(s)G(s)} T(s)$$

For $R(s) = 0$ and $N(s) = 0$.

Disturbance



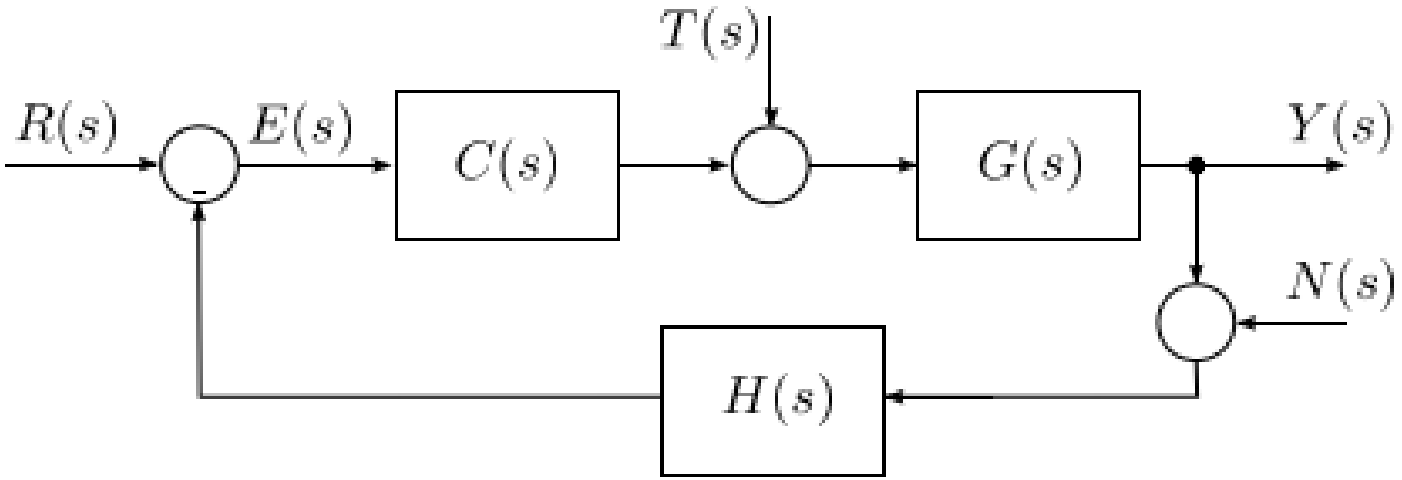
For $H(s) = 1$, $T(s) = 0$; and $N(s) = 0$, the output due to $R(s)$ is

$$Y_R(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} R(s)$$

For $H(s) = 1$, $T(s) = 0$, and $R(s) = 0$, the response to noise is

$$Y_N(s) = -\frac{C(s)G(s)}{1 + C(s)G(s)} N(s)$$

Disturbance and superposition

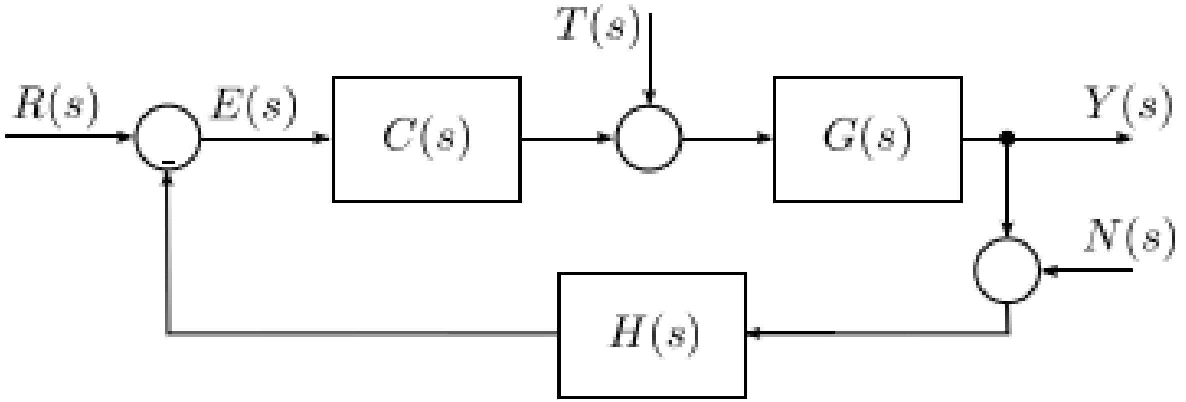


The principle of superposition gives the total response as

$$Y(s) = Y_R(s) + Y_N(s) + Y_T(s)$$

$$Y(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} R(s) + \frac{G(s)}{1 + C(s)G(s)} T(s) - \frac{C(s)G(s)}{1 + C(s)G(s)} N(s)$$

Disturbance and superposition



Defining the error as $E(s) = R(s) - Y(s)$ gives

$$E(s) = R(s) - \frac{C(s)G(s)}{1 + C(s)G(s)}R(s) - \frac{G(s)}{1 + C(s)G(s)}T(s) + \frac{C(s)G(s)}{1 + C(s)G(s)}N(s)$$

$$E(s) = \left(1 - \frac{C(s)G(s)}{1 + C(s)G(s)}\right)R(s) - \frac{G(s)}{1 + C(s)G(s)}T(s) + \frac{C(s)G(s)}{1 + C(s)G(s)}N(s)$$

$$E(s) = \frac{1}{1 + C(s)G(s)}R(s) - \frac{G(s)}{1 + C(s)G(s)}T(s) + \frac{C(s)G(s)}{1 + C(s)G(s)}N(s)$$

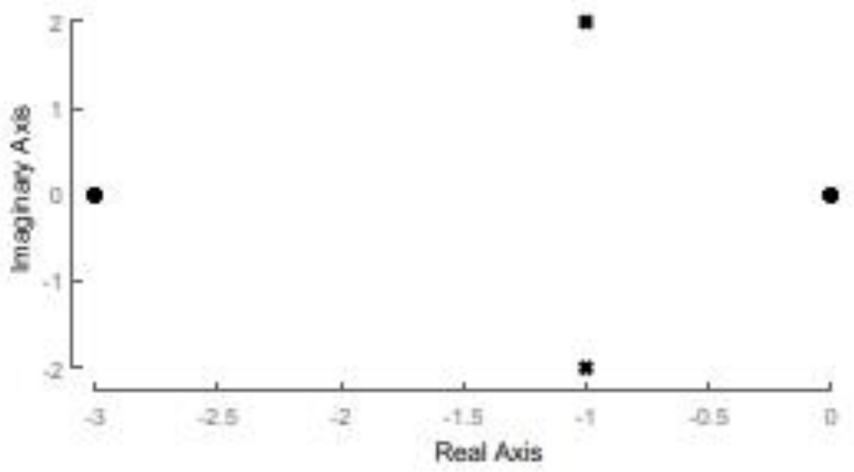
Poles and zeros

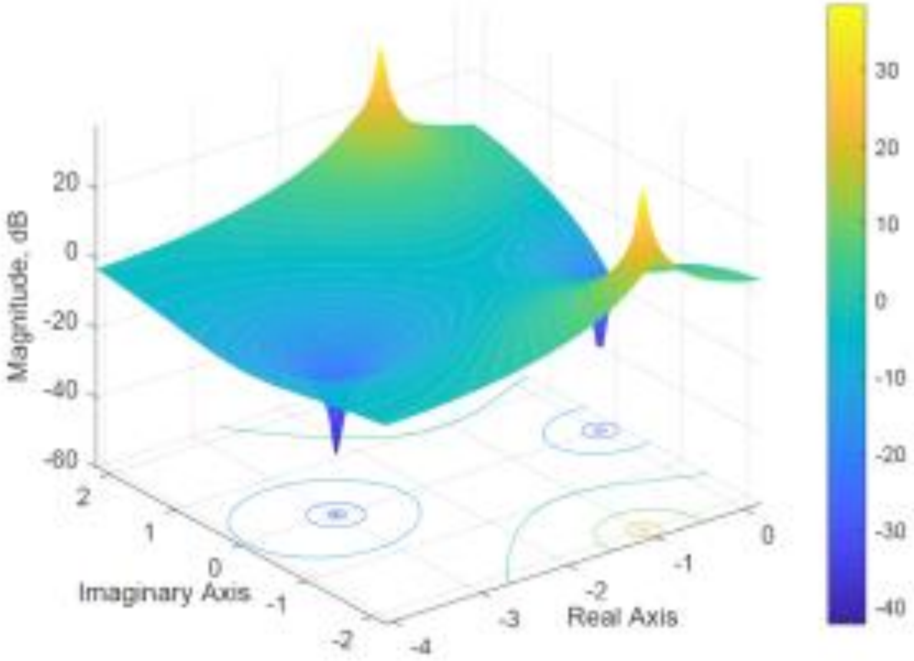
Consider the following function:

$$F(s) = \frac{s(s+3)}{s^2+2s+5}$$

$\rightarrow$ Poles: $-1+2j$, $-1-2j$

$\rightarrow$ Zeros: 0, -3





Disturbance rejection

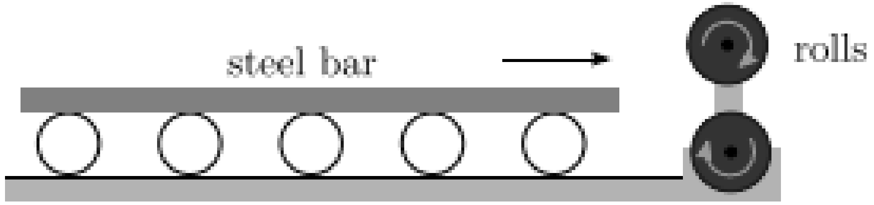
$$E(s) = \frac{1}{1 + C(s)G(s)} R(s) - \frac{G(s)}{1 + C(s)G(s)} T(s) + \frac{C(s)G(s)}{1 + C(s)G(s)} N(s)$$

To reduce the influence of the disturbance:

→ $C(s)$ must be large to minimize the influence of $T(s)$

→ $C(s)$ must be small to minimize the influence of $N(s)$

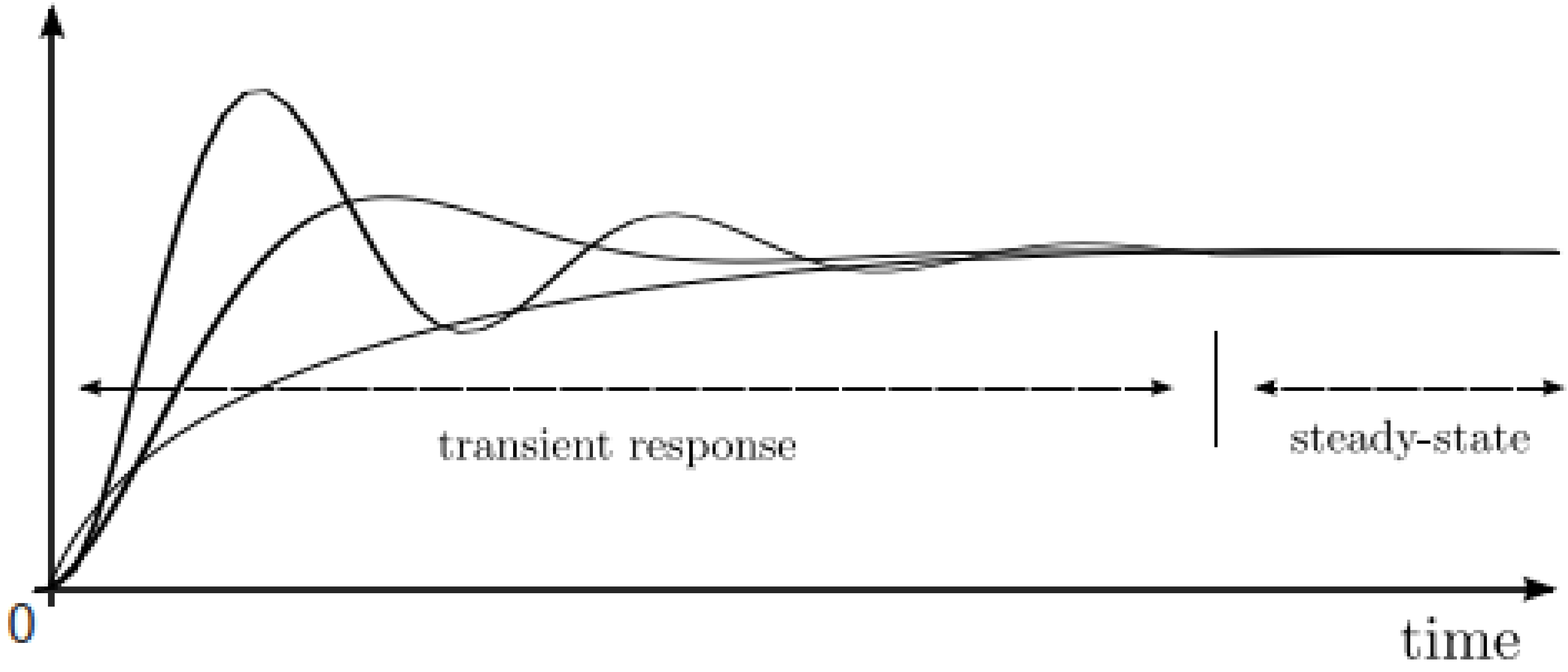
How to solve this conflict?



→ Make $C(s)$ large at low frequencies

→ Make $C(s)$ small at high frequencies

Transient and steady-state response



Steady-state error

If $N(s) = T(s) = 0$, the error is:

$$E(s) = \frac{1}{1 + C(s)G(s)} R(s)$$

The steady state error can be obtained using the final value theorem:

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

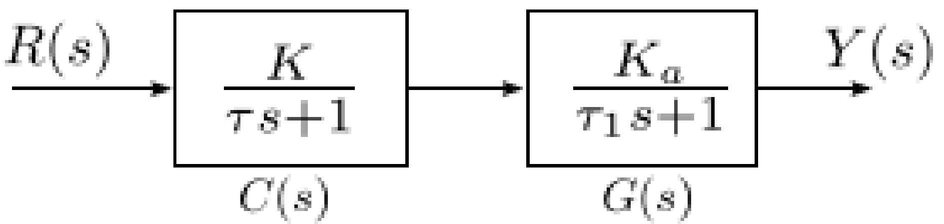
For the closed loop system (step input):

$$e_c(\infty) = \lim s \left(\frac{1}{1 + C(s)G(s)} \right) \left(\frac{1}{s} \right) = \left(\frac{1}{1 + C(0)G(0)} \right)$$

$C(0)G(0)$ is called the "DC gain".

Open-loop *vs* closed-loop

Open-loop: The error is $E(s) = R(s) - Y(s)$

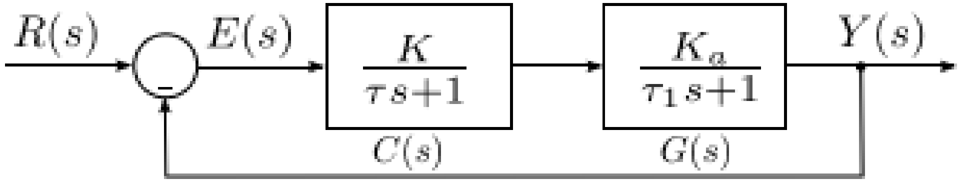


$$E(s) = R(s) - \frac{KK_a}{(\tau s + 1)(\tau_1 s + 1)} R(s)$$

What is the steady-state error?

Open-loop *vs* closed-loop

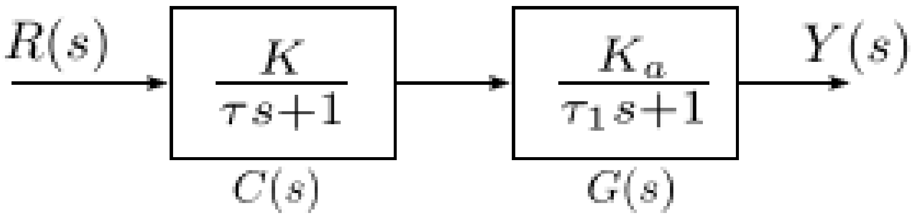
Closed-loop: The error is $E(s) = R(s) - Y(s)$



$$E(s) = R(s) - \frac{KK_a}{(\tau s + 1)(\tau_1 s + 1) + KK_a} R(s)$$

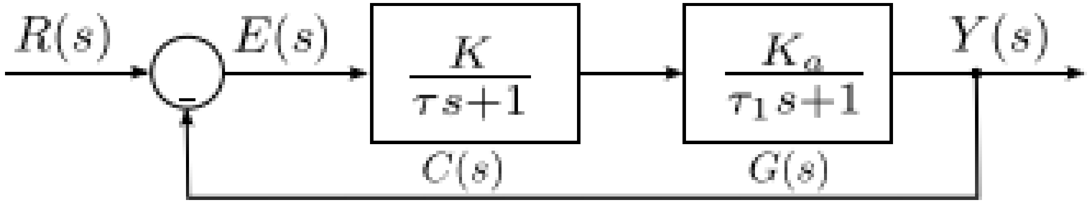
What is the steady-state error?

Open-loop *vs* closed-loop



Open-loop: The steady error is

$$E(s) = 1 - KK_a$$



Closed-loop: The steady error is

$$E(s) = \frac{1}{1 + KK_a}$$

Exercise 30

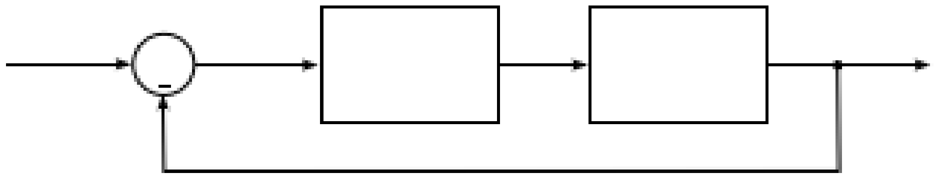
A robotic arm and camera are used to pick fruit as shown in the figure. The camera is used to close the feedback loop to a micro-controller, which controls the arm. The transfer function of the process is

$$G(s) = \frac{1}{(s+10)^2}$$

and the controller is a proportional gain so that $C(s) = K$. Calculate the steady-state error of the gripper for a step command A as a function of K.

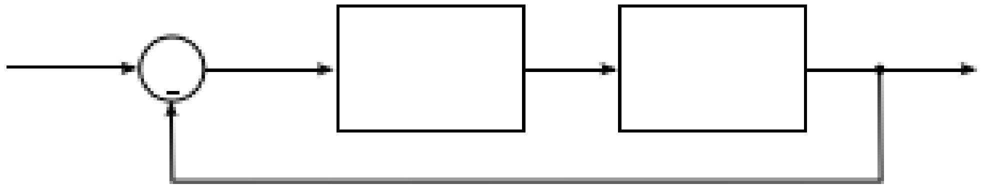
Exercise 30 - continued

$$G(s) = \frac{1}{(s+10)^2}, \qquad C(s) = K, \qquad R(s) =$$



Exercise 30 - continued

$$E(s) = \left(\frac{A}{s}\right)\frac{1}{1 + K/(s+10)^2}$$

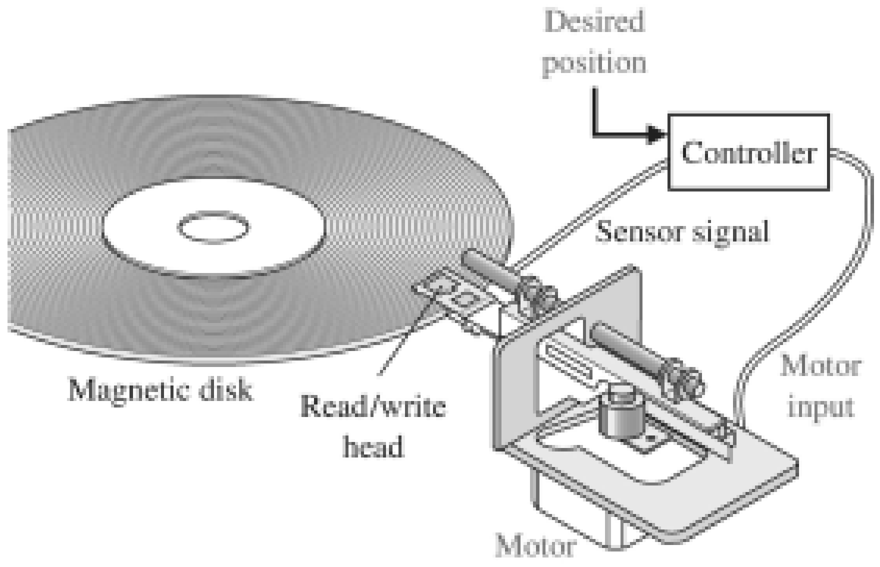


Exercise 31

A hard drive requires a motor to position a read/write head over the tracks of data on a spinning disk. The motor and head have the transfer function

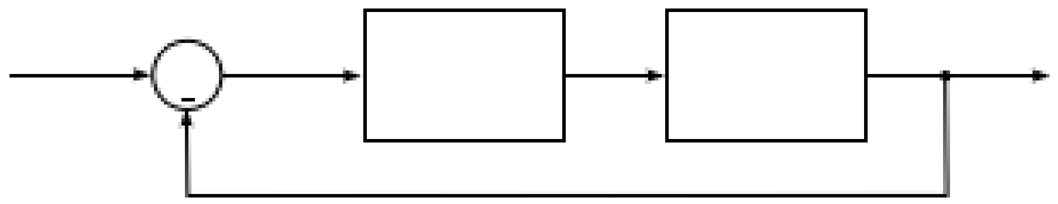
$$G(s) = \frac{100}{s(0.001s + 1)}$$

The controller is $C(s) = K$. Calculate K that yields a steady-state error of 0.1 mm for a ramp input of 10 cm/s.



Exercise 31 - continued

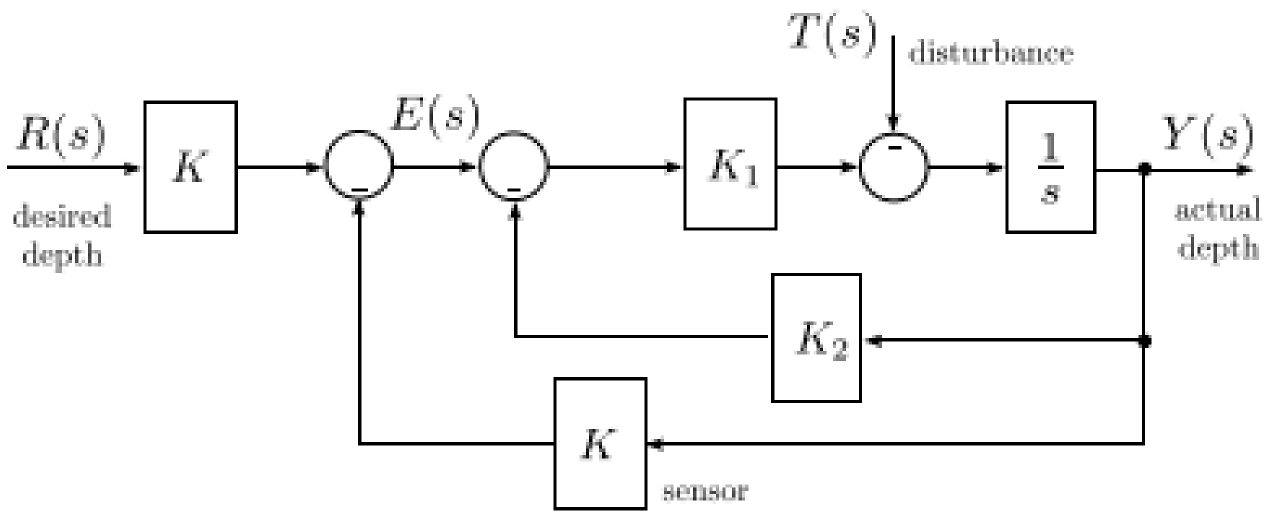
$$G(s) = \frac{100}{s(0.001s+1)}, \qquad C(s) = K, \qquad R(s) =$$



Exercise 31 - continued

Exercise 32

A submarine has a depth control system as illustrated.



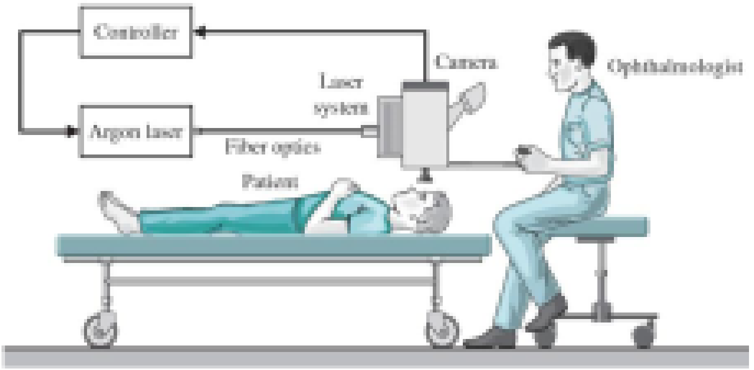
(a) Determine the transfer function $F(s) = Y(s)/R(s)$

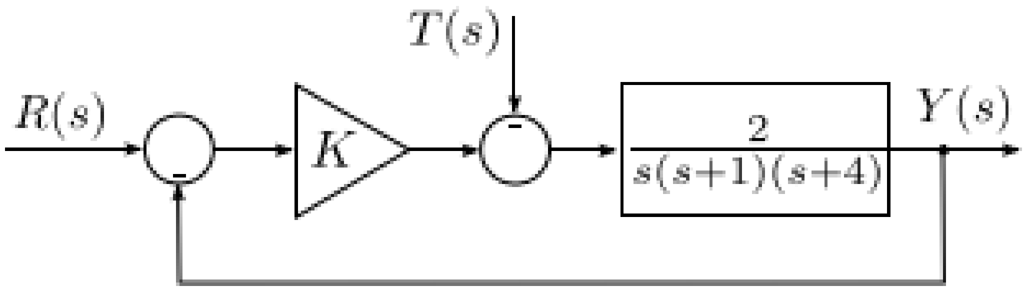
(b) Determine the steady state error for $T(s) = 1/s$ and $R(s) = 0$

(c) Calculate the response $y(t)$ for $R(s) = 1/s$ when $K = K_2 = 1$ and $1 < K < 10$. Select K_1 for the fastest response.

Exercise 33 - Design problem

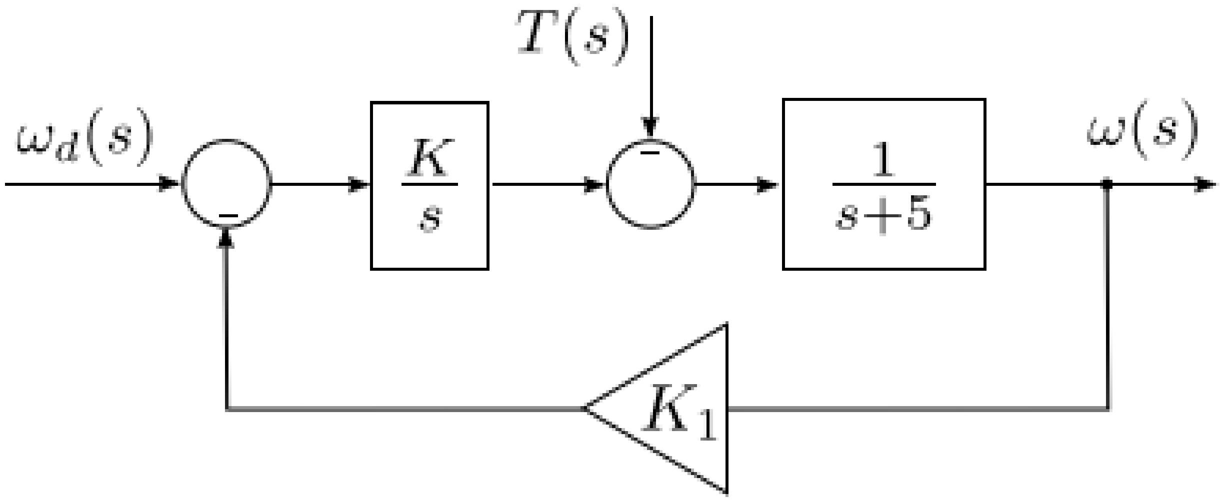
The position control system of a laser system for ophthalmologic surgery is shown in the figure. Select the gain K to ensure an appropriate transient response to a step change in $R(s)$ and the effect of disturbance is minimized. The steady-state error must be zero. The system is stable provided that $K < 10$. Plot the step response of the system for $1 < K < 15$ using Matlab.





Exercise 34 - Design problem

Consider the speed control system shown.



Determine:

(a) The range of K_1 so that the steady state tracking error is $e \leq 1\%$

(b) KK_1 so that the steady state error for $T(t) = 2t$ mrad/s for $0 \leq t \leq 5$ sec is less than 0.1 mrad/s.